Centers and limit cycles in polynomial systems of ODEs

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COURSE OBJECTIVES

The objectives of this course are to present efficient methods based on algorithms of computational commutative algebra for investigation of the following problems in the theory of autonomous polynomial systems of ODEs:
• the problem of distinguishing between a center and a focus
• the isochronicity problem
• the problem of small-amplitude limit cycles bifurcations (the cyclicity problem)
• the problem of bifurcations of critical periods.

COURSE DESCRIPTION

1^{st} lecture. First we give a short overview of the course. Then we present a short introduction to basic algorithms of computational algebra aimed to the study of polynomial ideals and their varieties. Groebner bases will be defined, few main algorithms in the theory of polynomial ideals will be presented. Methods to solve systems of algebraic polynomials (find decomposition of affine varieties) will discussed, in particular, an approach based on making use of modular arithmetic.

2^{nd} lecture. The center-focus problem and the problem of isochronicity and their connection to problems of integrability and linearizability will be discussed. Algorithms to compute necessary conditions for integrability and isochronicity will be presented. Methods to construct first integrals and linearizing substitutions or to prove their existence will be given. A generalization of the center problem to higher dimensional systems will be discussed.

3^{rd} lecture. Time-reversibility of polynomial systems with respect to linear transformations will be studied. An algorithm to find all time-reversible systems inside of a given family of polynomial systems will be presented. An interconnection of time-reversibility and invariants of the rotation group will be discussed.

4^{th} lecture. An approach to study small-limit cycles bifurcations (cyclicity) of polynomial systems will be presented. It will be demonstrated that the problem can be reduced

to the algebraic problem of finding a basis of a certain polynomial ideal, called the Bautin ideal of the system. In the case when the ideal is radical we show that the problem can be easily solved using algorithms of computational algebra.

5^{th} lecture. We discuss an approach to study the cyclicity problem in the case, when the Bautin ideal is non-radical. A method to study cyclicity of each component of the center variety will be presented. The problem of bifurcations of critical periods will be stated and its connection to the cyclicity problem will be discussed.

The needed background to follow the course is acquired in algebra and differential equations courses taught at undergraduate level.

REFERENCES:

1. C. Christopher and C. Li. *Limit Cycles of Differential Equations.* Basel: Birkhäuser–Verlag, 2007.

2. D. Cox, J. Little, and D. O'Shea. *Ideals, Varieties, and Algorithms.* New York: Springer–Verlag, 1992.

3. V. G. Romanovski and D. S. Shafer. *The Center and Cyclicity Problems: A Computational Algebra Approach.* Boston: Birkhäuser, 2009.

4. Trends in Mathematics, Differential Equations with Symbolic Computations (D. Wang and Z. Zheng, Eds.) Basel: Birkhauser–Verlag, 2005.