BILIPSCHITZ GEOMETRY OF COMPLEX SINGULARITIES

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The course will present a bilipschitz point of view on the geometry of a normal complex surface X in a neighbourhood of a singular point $p \in X$. It is based on a joint work with Lev Birbrair and Walter Neumann.

It is well known that for all sufficiently small $\epsilon > 0$ the intersection of X with the sphere S_ϵ^{2n-1} of radius ϵ about p is transverse, and X is therefore locally "topologically conical," i.e., homeomorphic to the cone on its link $X \cap S_\epsilon^{2n-1}$. However, as shown by Birbrair and Fernandez, (X, p) need not be "metrically conical", i.e. bilipschitz equivalent to a standard metric cone when X is equipped with the Riemanian metric (the so-called inner metric) induced by the ambient space. In fact, it was shown by Birbrair, Fernandez and Neumann that it rather rarely is.

I will present a complete classification of the bilipschitz geometry of (X, p). It starts with a decomposition of a normal complex surface singularity into its "thick" and "thin" parts. The former is essentially metrically conical, while the latter shrinks rapidly in thickness as it approaches the origin. The thin part is empty if and only if the singularity is metrically conical. Then the complete classification consists of a refinement of the thin part into geometric pieces.

I will also present some results on the outer metric of a normal surface, which is the metric induced on (X, p) by the hermitian metric. In particular, I will show a relation between two points of view on equisingularity : Zariski's and bilipschitz equisingularity.

I will also present a list of open problem related with this new point of view on classifying complex singularities.