12th International Workshop on Real and Complex Singularities

Celebrating the 60th birthday of Shyuichi Izumiya

22nd-27th July 2012

and

School on Singularity Theory

16th-21st July 2012

Book of Abstracts

ICMC-USP São Carlos, Brazil 16th-27th July 2012

Organizing Committee

Victor Hugo Jorge Perez (ICMC-USP-Coordinator)
Raimundo Nonato Araújo dos Santos (ICMC-USP-Coordinator)
Roberto Callejas Bedregal (UFPB)
Fabio Scalco Dias (UNIFEI)
Miriam Garcia Manoel (ICMC-USP)
Ana Claudia Nabarro (ICMC-USP)
Regilene Oliveira (ICMC-USP)
Bruna Orefice (UFSCAR)
Raul Oset Sinha (ICMC-USP)
Masatomo Takahashi (Muroran Institute of Technology)
João Tomazella (UFSCAR)
Takashi Sano (Hokkai-Gakuen University)
Roberta Wik-Atique (ICMC-USP)

Scientific Committee

Lev Birbrair (Federal University of Ceará)
Maria del Carmen Romero Fuster (University of Valencia)
Goo Ishikawa (Hokkaido University)
Abramo Hefez (Federal University of Fluminense)
Stanislaw Janeczko (Polish Academy of Sciences and Warsaw University of Technology)
Maria Aparecida Soares Ruas (ICMC-USP)
Osamu Saeki (Kyushu University)
Marcio Soares (Federal University of Minas Gerais)
David Trotman (University of Marseille)
Masaaki Umehara (Tokyo Institute of Technology)

Contents

1. Introduction

The *Workshops on Real and Complex Singularities* form a series of biennial meetings organized by the Singularities group at Instituto de Ciências Matemáticas e de Computação of São Paulo University (ICMC-USP), Brazil. Their main purpose is to bring together world experts and young researchers in singularity theory, applications and related fields to report recent achievements and exchange ideas, addressing trends of research in a stimulating environment.

The twelfth edition, to be held from the 22nd to the 27th of July of 2012, is celebrating the 60th birthday of professor Shyuichi Izumiya. This time, prior to the workshop, a School on Singularity theory has also been organized, to be held from the 16th to the 21st of July. The present book is the collection of titles and abstracts of the mini-courses, plenary talks, parallel sessions, short talks and posters that shall be presented during these two weeks.

2. School on Singularity Theory, 16th-21st July 2012

2.1. Mini-Courses.

Non-isolated singularities

Lê Dung Trang (ICTP, Italy)

Abstract. First, we will review general results on isolated singularities and stratifications. Then, we shall concentrate on Whitney stratifications and different ways to characterize them. If, there is time left, we shall describe good properties of an equisingularity stratification and related open problems.

References:

1. Lê Dung Trang; Teissier, B. Cycles evanescentes, sections planes et conditions de Whitney. II, in Singularities, Part 2 (Arcata, Calif., 1981), 65103, Proc. Sympos. Pure Math., 40, Amer. Math. Soc., Providence, RI, 1983
2. Lê Dung Trang; Teissier, Bernard Limites d'espaces tangents en géométrie analytique, Comment. Math. Helv. 63 (1988), no. 4, 540578.

Groebner bases and beyond - An introduction to the CAS Singular

Hans Schnemann (University of Kaiserslautern, Germany)

Abstract. Germany Groebner bases are the main theoretical tool available for the symbolic manipulation of non-linear systems of equations. These appear in a wide variety of applications. In first place, however, Groebner bases have become an indispensable supporting tool in the study of algebraic geometry. This course will introduce the students to the basics of the theory, with a view towards its applications. The course will also give an introduction to the computer algebra system SINGULAR. We will discuss fundamental algorithms (Groebner bases, Groebner bases for non-commutative rings, for free modules, syzygies and free resolutions, etc.), their implementation in SINGULAR, and a number of applications to problems of theoretical and practical interest.

On singularities of maps

David Mond (University of Warwick, UK)

Abstract. The aim is to introduce graduate students to methods and problems in the study of singularities of mappings. Lectures will cover

1. Examples, and basic terms: critical points and critical values, Thom-Boardman singularities. Basics of analytic geometry - though without systematic use of sheaves; normal and non normal varieties.
2. Notion from commutative algebra: dimension, depth and Cohen-Macaulay rings.
3. Generic behavior, jet bundles and Thom's transversality theorem.
4. Left-right equivalence of analytic map-germs, left-right tangent spaces, and Damon's Theorem identifying left-right equivalence with $\mathcal{K}_D$-equivalence of sections of stable discriminants.
5. Use of infinitesimal methods to prove finite determinacy and versality theorems.
6. The geometry and topology of deformations: conservation of multiplicity and μ versus τ .
7. Multiple points by means of Fitting ideals (in the target) and by residual intersections (in the source).
8. Free divisors and logarithmic vector fields and differential forms.

I will provide exercises, and will run problem classes if students want.

Centers and limit cycles in polynomial systems of ODEs

Valery Romanovsky (University of Maribor, Slovenia)

Course objectives.

The objectives of this course are to present efficient methods based on algorithms of computational commutative algebra for investigation of the following problems in the theory of autonomous polynomial systems of ODEs:

1. the problem of distinguishing between a center and a focus;
2. the isochronicity problem;
3. the problem of small-amplitude limit cycles bifurcations (the cyclicity problem);
4. the problem of bifurcations of critical periods.

Course description

1st lecture. First we give a short overview of the course. Then we present a short introduction to basic algorithms of computational algebra aimed to the study of polynomial ideals and their varieties. Groebner bases will be defined, few main algorithms in the theory of polynomial ideals will be presented. Methods to solve systems of algebraic polynomials (find decomposition of affine varieties) will discussed, in particular, an approach based on making use of modular arithmetic.

2nd lecture. The center-focus problem and the problem of isochronicity and their connection to problems of integrability and linearizability will be discussed. Algorithms to compute necessary conditions for integrability and isochronicity will be presented. Methods to construct first integrals and linearizing substitutions or to prove their existence will be given. A generalization of the center problem to higher dimensional systems will be discussed.

3rd lecture. Time-reversibility of polynomial systems with respect to linear transformations will be studied. An algorithm to find all time-reversible systems inside of a given family of polynomial systems will be presented. An interconnection of time-reversibility and invariants of the rotation group will be discussed.

4th lecture. An approach to study small-limit cycles bifurcations (cyclicity) of polynomial systems will be presented. It will be demonstrated that the problem can be reduced 1 to the algebraic problem of finding a basis of a certain polynomial ideal, called the Bautin ideal of the system. In the case when the ideal is radical we show that the problem can be easily solved using algorithms of computational algebra.

5th lecture. We discuss an approach to study the cyclicity problem in the case, when the Bautin ideal is non-radical. A method to study cyclicity of each component of the center variety will be presented. The problem of bifurcations of critical periods will be stated and its connection to the cyclicity problem will be discussed. The needed background to follow the course is acquired in algebra and differential equations courses taught at undergraduate level.

References:

1. C. Christopher and C. Li. Limit Cycles of Differential Equations. Basel: Birkhäuser–Verlag, 2007.
2. D. Cox, J. Little, and D. O'Shea. Ideals, Varieties, and Algorithms. New York: Springer–Verlag, 1992.
3. V. G. Romanovski and D. S. Shafer. The Center and Cyclicity Problems: A Computational Algebra Approach. Boston: Birkhäuser, 2009.
4. Trends in Mathematics, Differential Equations with Symbolic Computations (D.Wang and Z. Zheng, Eds.) Basel: Birkhäuser–Verlag, 2005.

Topology and geometry of real singularities

Nicolas Dutertre (Aix-Marseille Université, France)

Abstract. The aim of this mini-course is to provide the students with tools and techniques used in the theory of real singularities and to apply them to get interesting results on the topology and geometry of real singularities. The content of the course will be the following :

1. Background in differential topology and differential geometry;
2. Basic results on the topology of real analytic/semi-analytic sets;
3. Methods for the computation of topological invariants of real analytic/semianalytic sets;
4. Results on the geometry of real analytic/semi-analytic sets.

2.2. Poster.

There will be a poster session during the School. The titles and abstracts will be announced throughout the week. These abstracts can be found amongst the abstracts in Section 4.4 Poster Session.

3. Brazilian and Japanese young researchers meeting

Atsushi Yano (Hokkaido University)

Title: **On Monge characteristic systems of hyperbolic differential systems**

Abstract: **See abstract in Workshop Posters**

Daisuke Nakajo (Kyushu University)

Title: **Indefinite improper affine spheres and indefinite affine minimal surfaces with singularities**

Abstract: **See abstract in Workshop Posters**

Guillermo Peñafort Sanchis (Universitat De Valencia)

Title: **Invariants and Marar-Mond formulas for double folds and Coxeter maps**

Abstract: We extend some properties and methods related to the family of fold maps $\mathbb{K}^2 \to \mathbb{K}^3$, $(x, y) \mapsto (x, y^2, f(x, y))$ to a more general family, namely the Coxeter maps $\mathbb{K}^n \to \mathbb{K}^{n+1}$, $(x, y) \mapsto (\alpha(x, y), f(x, y))$, where α is invariant under the action of a Coxeter group. We describe the double point space of Coxeter maps, find candidates for $\mathcal{A}$-invariants and show an equivalence relation between Coxeter maps which may be closely related to the study of $\mathcal{A}$-equivalence. We emphasize on double folds, where $\alpha(x, y) = (x^2, y^2)$ and give for them an adaptation of Marar- Mond formulas and the description of their links for the real case.

Iris de Oliveira (ICMC-USP)

Title: **Reversible-equivariant Belitskii normal form**

Abstract: **See abstract in Workshop Posters**

Kazuto Takao (Osaka University)

Title: **Two Morse functions and singularities of the product map**

Abstract: **See abstract in Workshop Posters**

Luis Renato G. Dias (ICMC-USP)

Title: **Atypical values and regularity conditions at infinity**

Abstract: We describe the relation between four different types of regularity conditions which have been used in the literature in order to control the asymptotic behaviour of semi-algebraic mappings. We prove a new Morse-Sard type theorem for the asymptotic critical values of semi-algebraic mappings. For polynomial mappings, we also present some algebraic characterizations of regularity at infinity."

Luis Florial Espinoza Sánchez (ICMC–USP)

Title: **Surfaces in $\mathbb{R}^4$ from the affine differential geometry viewpoint**

Abstract: In [2], Little described the local second order invariants of compact surfaces in Euclidean 4-space and Mochida, Romero-Fuster and Ruas in [3], obtain results on the geometry of surfaces in Euclidean 4-space from analysis of their generic contacts with hyperplanes. They gave a geometrical characterization to the singularities of the height functions on such manifolds and showed that the inflection points are the umbilics of these functions. In this work we consider surfaces in $\mathbb{R}^4$ from the affine differential geometry viewpoint. We define the concepts of affine binormal and affine asymptotic directions for nondegenerate affine-immersed surfaces in affine 4-space using affine metric and the problem of generalized eigenvalues and eigenvectors, we compute these objects to the product of two planar curves and surfaces in the Monge form. We characterize the affine asymptotic lines by a differential equation similarly to the Euclidean case.

References:

1. Declan Davis. Affine diferential geometry and singularity theory. PhD thesis, University of Liverpool, April 2008.
2. John A. Little. On singularities of submanifolds of higher dimensional Euclidean spaces. Ann. Mat. Pura Appl. (4), 83:261–335, 1969.
3. Dirce Kiyomi Hayashida Mochida, Maria Del Carmen Romero Fuster, and Maria Aparecida Soares Ruas. The geometry of surfaces in 4-space from a contact viewpoint. Geom. Dedicata, 54(3):323–332, 1995.

Mirjana Milijevic (Hokkaido University)

Title: **CR submanifolds**

Abstract: **See abstract in Workshop Posters**

Na Hu (Hokkaido University)

Title: **Affine space curves and homogeneous surfaces**

Abstract: **See abstract in Workshop Posters**

Pedro Henrique Apoliano Albuquerque Lima (ICMC-USP)

Title: **Properties of fiber cone to good filtration**

Abstract: Let $(A, \mathfrak{m})$ be a Noetherian local ring and $\mathfrak{F} = (I_n)_{n\geq 0}$ a filtration. In this presentation, we will speak about the Gorenstein properties of the fiber cone $F(\mathfrak{F})$, where $\mathfrak{F}$ is a Hilbert filtration. Suppose that $F(\mathfrak{F})$ and $G(\mathfrak{F})$ are Cohen-Macaulay. If in addition, the associated graded ring $G(\mathfrak{F})$ is Gorenstein; similarly to the I-adic case, we obtain a necessary and sufficient condition, in terms of lengths and minimal number of generators of ideals, for Gorensteiness of the fiber cone. Moreover, we find a description of the canonical module of $F(\mathfrak{F})$ and show that even in the Hilbert filtration case, the multiplicity of the canonical module of the fiber cone is upper bounded by multiplicity of the canonical modules of the associated graded ring

Joint work with: *V. H. Jorge Pérez.*

Takami Sato (Hokkaido University)

Title: **Curves on a spacelike surface in Lorentz-Minkowski 3-space**

Abstract: **See abstract in Workshop Posters**

Wataru Yukuno (Hokkaido University)

Title: **Generic properties of singular trajectories of control affine system**

Abstract: **See abstract in Workshop Posters**

Yang Jiang (Hokkaido University)

Title: **Lightcone dualities for curves in the sphere**

Abstract: **See abstract in Workshop Posters**

Yusuke Mizota (Kyushu University)

Title: **Constructing generators for the module of liftable vector fields**

Abstract: **See abstract in Workshop Posters**

---

4. 12th International Workshop on Real and Complex Singularities

4.1. **Mini-Courses.**

Bilipschitz geometry of complex singularities

Anne Pichon (Aix-Marseille Université)

Abstract. The course will present a bilipschitz point of view on the geometry of a normal complex surface X in a neighbourhood of a singular point $p \in X$. It is based on a joint work with Lev Birbrair and Walter Neumann.

It is well known that for all sufficiently small $\epsilon > 0$ the intersection of X with the sphere S^{2n-1}_{ϵ} of radius ϵ about p is transverse, and X is therefore locally "topologically conical," i.e., homeomorphic to the cone on its link $X \cap S^{2n-1}_{\epsilon}$. However, as shown by Birbrair and Fernandez, (X, p) need not be "metrically conical", i.e. bilipschitz equivalent to a standard metric cone when X is equipped with the Riemanian metric (the so-called inner metric) induced by the ambient space. In fact, it was shown by Birbrair, Fernandez and Neumann that it rather rarely is.

I will present a complete classification of the bilipschitz geometry of (X, p). It starts with a decomposition of a normal complex surface singularity into its "thick" and "thin" parts. The former is essentially metrically conical, while the latter shrinks rapidly in thickness as it approaches the origin. The thin part is empty if and only if the singularity is metrically conical. Then the complete classification consists of a refinement of the thin part into geometric pieces.

I will also present some results on the outer metric of a normal surface, which is the metric induced on (X, p) by the hermitian metric. In particular, I will show a relation between two points of view on equisingularity : Zariski's and bilipschitz equisingularity.

I will also present a list of open problem related with this new point of view on classifying complex singularities.

Singularities and characteristic classes for differentiable maps.

Toru Ohmoto (Dept. Math., Hokkaido University)

Abstract. This is a crash course on variants of *Thom polynomials* (Tp) for singularities of real and complex maps. By definition, Tp is just a universal expression of the fundamental class of the locus of singularities of maps with a prescribed type, and its origin goes back to R. Thom's talk in Strasbourg around 1957. The Tp theory has potential applications to the enumerative geometry from classics to modern in both contexts of algebraic geometry and differential topology (to say, currently, symplectic topology).

First we reivew a basic material of the theory for non-experts, especially graduate students. Then I'm planning to enter two different topics. The former one is *higher Thom polynomial*: we discuss in algebro-geometric

context a universal expression of the Chern-Schwartz-MacPherson class of singular loci or discriminants of complex maps as a higher degree generalization of Tp. There are many interesting applications, e.g. for the Milnor number μ, and in a sense it relates to an equivariant version of the Milnor class. The second topic is about real singularities: we discuss local Vassiliev-type invariants for generic C^∞ maps in relation with Tp theory. I will mention a number of open problems.

4.2. **Plenary Talks.**

On the topology of the polar curve

Abramo Hefez (Universidade Federal Fluminense)

Abstract. We will present some results concerning the topology of the polar curve of an irreducible plane curve singularity varying in an equisingularity class, taking into account its analytical type.

Joint work with: *Mauro Fernando H. Iglesias and Marcelo E. Hernandes*

On choking horns in complex algebraic singularities

Alexandre Fernandes (Universidade Federal Do Ceará)

Abstract. I will talk about a joint work with Lev Birbrair, Vincent Grandjean and Donal O'Shea where we study some aspects of Lipschitz Geometry of Complex Algebraic Singularities. We introduce a notion of fast contracting cycles which are cycles on the family of the t-sections of an algebraic variety by the small spheres of radius t such that they cannot be boundaries of chains, in those spheres, with diameter vanishing faster than linearly . We use these fast cycles to show that classical singularities are not metrically conical. Moreover, we prove that there are infinitely many singular sets who are locally homeomorphic but not locally bi-Lipschitz homeomorphic with respect the inner metric.

On the monodromy of regular maps from an affine surface to a curve

Aryampilly Jayanthan Parameswaran (Tata Institute of Fundamental Research)

Abstract. Let X be a quasi projective, connected, nonsingular surface and let $f : X \to C$ be a regular map onto a nonsingular curve. Let $\bar{f} : \bar{X} \to \bar{C}$ be a compact resolution of the indeterminacy points at infinity, namely $\bar{X}$ and $\bar{C}$ are nonsingular and such that $\bar{X}$ mX is a connected normal crossing divisor.

We study here the monodromy of the map f in relation to that of $\bar{f}$.

Joint work with: *M. Tibar*

Lojasiewicz exponents and its relation with other analytic invariants

Carles Bivià-Ausina (Universitat Politècnica de València)

Abstract. We show recent advances concerning the notion of Łojasiewicz exponent of a set of ideals. In particular, we relate these invariants with bi-Lipschitz equivalence of functions and we show some relations between Łojasiewicz exponents and log canonical thresholds of ideals. This is a joint work with Toshizumi Fukui (Saitama University).

Joint work with: *Toshizumi Fukui (Saitama University)*

Shyuichi Izumiya's mathematical works

Goo Ishikawa (Hokkaido University)

Abstract. I will explain some of Shyuichi Izumiya's mathematical works, emphasizing their significance and influence of his important mathematical contributions in singularity theory and related areas.

Nash conjecture on arc spaces.

Javier Fernandez de Bobadilla (CSIC)

Abstract. In a joint work with M. Pe Pereira, the speaker has proved J. Nash conjecture on the correspondence between the irreducible components of the space of arcs centered at the singular set of an algebraic or analytic surface and the essential components of its resolution of singularities. Counterexamples to the same question in dimension 4 and higher were given by S. Ishii and J. Kollar, and recently T. de Fernex has given a 3-dimensional counterexample. In the talk we will explain the proof for the surface case and briefly discuss the higher dimensional counterexamples.

Rational maps preserving webs

Jorge Vitório Pereira (IMPA)

Abstract. I will describe the structure of rational self-maps of projective surfaces preserving webs. Most of the talk will be based on joint work with C. Favre.

On the topology of complete intersection germs on singular varieties

Jose Seade Kuri (Instituto De Matematicas, Universidad Nacional Autonoma De Mexico)

Abstract. We extend the classical Lê-Greuel formula for complete intersection germs, to the case of functions defined on singular varieties.

Joint work with: *Roberto Callejas-Bedregal, Michelle Morgado, Marcelo Saia*

Slicing corank 1 map germs from $\mathbb{C}^2$ to $\mathbb{C}^3$

Juan José Nuño-Ballesteros (Universitat De Valencia)

Abstract. The classification and the geometry of corank 1 map germs $f : (\mathbb{C}^2, 0) \to (\mathbb{C}^3, 0)$ have been studied by David Mond. Normal forms of such maps $f(x, y) = (x, p(x, y), q(x, y))$ suggest, at least in some cases, that they could be seen as 1-parameter unfoldings of the plane curve $\gamma(y) = (p(0, y), q(0, y))$. The plane curve γ will be called the transverse slice of the map germ f and we show how to count its codimension 1 singularities. We shall then compare invariants of f with those of the associated transverse slice. From this point of view we reinterpret Mond's geometric results and we obtain a deep understanding of the rich geometry of such map germs.

Joint work with: *W.L. Marar*

TBA

Lê Dung Trang (ICTP-Italy)

Abstract. TBA

Multiplicities of modules and Newton polyhedra

Marcelo José Saia (ICMC–USP)

Abstract. The computation of the Hilbert-Samuel multiplicity of an ideal of finite co-length is one of the main tools used to calculate geometric invariants of singularities and it is a challenging problem in commutative algebra to show methods which alow us to compute such multiplicity.

In a general set-up, Buchsbaum and Rim in gave a generalization of the Samuel multiplicity of $\mathfrak{m}$-primary ideals for the case of finite co-length submodules of a free module R^p, where $(R, \mathfrak{m})$ is a d-dimensional Noetherian local ring. Kirby and Rees showed how to characterize the Buchsbaum-Rim multiplicity of submodules M in R^p given as a direct sum of $\mathfrak{m}$-primary ideals $M = I_1 \bigoplus \ldots \bigoplus I_p$ in R, in terms of the mixed multiplicities of these ideals as defined by Teissier.

One question is to extend the result of Kirby and Rees and show how to compute the Buchsbaum-Rim multiplicity of any submodule M of finite co-length in R^p in terms of appropriate multiplicities associated to a finite set of sub modules M_1, M_2, …, M_s.

In this article we introduce a new set of multiplicities defined on any family of finite co-length modules $M_1, \ldots, M_s$, with each submodule M_i of finite co-length in R^{p_i} and show how to compute the Buchsbaum-Rim multiplicity of the finite co-length submodule of R^p, $M = M_1 \bigoplus \ldots \bigoplus M_s$ in terms of the $\star$-multiplicities associated to the family $M_1, \ldots, M_s$, generalizing the result of Kirby and Rees.

On the other side, the multiplicity is a concept that is strongly related to the the integral closure condition. The notion of integral closure of ideals was extended to modules by Rees, Kleyman-Thorup and Kirby-Rees. Moreover, Kirby and Rees showed how to characterize the Buchsbaum-Rim multiplicity of submodules M in R^p given as a direct sum of $\mathfrak{m}$-primary ideals $M = I_1 \bigoplus \ldots \bigoplus I_p$ in R, in terms of the mixed multiplicities of these ideals. Biviá-Ausina studies the integral closure and the Buchsbaum-Rim multiplicity of submodules M in $\mathcal{O}_n^p$. Using the generalization given here for the result of Kirby-Rees we apply the results of Biviá-Ausina to characterize the multiplicities defined here in terms of volumes of appropriate Newton polyhedra, under a Newton non-degeneracy condition for the submodules M and M_i.

Joint work with: *V H Jorge Perez and R Callejas Bedregal*

Differential geometry of surfaces with singularities

Masaaki Umehara (Tokyo Institute of Technology)

Abstract. I would like to talk on maximal surfaces with singularities in R^3_1, isometric deformations of cross caps in R^3, and the behavior of sectional curvature of hypersurfaces with A_k-singularities in R^n for $2 \leq k \leq n$.

Joint work with: *Kentaro Saji (Kobe Univ.) and Kotaro Yamada (Tokyo Insitiute of Technology)*

Invariant theory and relative symmetries of mappings

Miriam Manoel (ICMC–USP)

Abstract. Symmetries of a mapping form a group with a linear real action on the space of variables that commutes with the mapping. Relative symmetries extend this notion, forming a larger group Γ whose subset of symmetries is a normal subgroup H of finite index. The actions of Γ on source and target are now distinct. More precisely, consider the cyclic group $\mathbf{Z}_m$, where m is the index of H, and an epimorphism $\sigma : \Gamma \to \mathbf{Z}_m$; for $\gamma \in \Gamma$, if $x \mapsto \gamma x$ denotes the action on the source, then $x \mapsto \sigma(\gamma)\gamma x$ is the action on the target. In this talk, we discuss the power of algebraic tools from invariant theory to obtain the general form of mappings with relative symmetries, as well as for the systematic study of the singularities and bifurcations of vector fields under relative symmetries. In particular, we shall address more attention to $m = 2$, which is the case of equivariant and reversible vector fields. The results presented here are part of a series of joint works with P.H. Baptistelli.

Joint work with: *P.H. Baptistelli.*

The kite of a plane curve singularity

Patrick Popescu-Pampu (Lille 1)

Abstract. I will present joint work with Garcia Barroso and Gonzalez Perez. I have associated before a 2-dimensional simplicial complex, called its kite, to any constellation of infinitely near points of a smooth point of a surface. I have showed that this complex allows to understand geometrically the relation between the Enriques diagram of the constellation and the dual graph obtained by totally blowing it up. In our present work, we explain how to construct the kite of the constellation of the minimal embedded resolution of a plane curve singularity, starting from Puiseux series of the different irreducible components of the singularity.

Lê cycles and Milnor classes

Roberto Callejas-Bedregal (Universidade Federal da Paraíba-UFPB)

Abstract. The purpose of this work is to establish a link between the theory of Chern classes for singular varieties and the geometry of the varieties in question. Namely, we show that if Z is a hypersurface in a compact complex manifold, defined by the zero-scheme of a nonzero holomorphic section of a very ample line bundle, then its Milnor classes, regarded as elements in the Chow group of Z, determine the global Lê cycles of Z; and viceversa: The Lê cycles determine the Milnor classes. Morally this implies, among other things, that the Milnor classes determine the topology of the local Milnor fibres at each point of Z, and the geometry of the local Milnor fibres determines the corresponding Milnor classes.

Joint work with: *M. F. Z. Morgado and J. Seade*

Nash problem, arc spaces and applications

Shihoko Ishii (University of Tokyo)

Abstract. The arc space was introduced by J. F. Nash in his preprint in 1968 and in the paper he posed a problem, so called the Nash problem. In this talk, I will introduce the Nash problem, show the geometric properties of arc spaces and then show applications of arc spaces in algebraic geometry.

Gravitational lensing and crystalline caustics

Stanislaw Janeczko (Institute Of Mathematics, Polish Academy Of Sciences)

Abstract. Model of gravitational lensing is build by using the generalized canonical mappings between symplectic spaces. The focusing of matter is concentrated around caustics defined by Lagrangian submanifolds given by compositions of symplectic relations. The singularity theory of caustics in various geometrical models is revisited. We show that the crystalline shapes on the plane are classified by symplectic singularities of plane branches.

An Infinitesimal Approach to Bi-Lipschitz Stratifications

Terence Gaffney (Northeastern University)

Abstract. A theory of Bi-Lipschitz equisingularity is emerging from the work of many authors. Comparing the current state of the theory with that of Whitney equisingularity, there is no infinitesimal description of the stratification condition as there is for Whitney equisingularity. We describe an approach to this description using the theory of integral closure of modules, and report on progress to date and associated open questions.

4.3. **Parallel Sessions.**

Realizing homology classes by maps with mild singularities

Andras Szucs (Eotvos University)

Abstract. We show that for any $k > 1$ there is a Z_2 homology class of codimension k in a smooth manifold of sufficiently high dimension that can not be realized by an immersion. Similar result holds if we replace the word "immersion" by maps having multisingularities only from any fixed finite set of multisingularities.

Joint work with: *Mark Grant*

Aurélio Menegon Neto (UNAM)

Title: **TBA**

Abstract: **TBA**

The vanishing Euler characteristic of a determinantal variety germ and the Milnor number of a function

Bruna Oréfice Okamoto (Universidade Federal de São Carlos)

Abstract. Given $F : (\mathbb{C}^N, 0) \to M_{m,n}(\mathbb{C})$ a holomorphic function germ, let $(X, 0)$ be the isolated determinantal singularity given by $X = F^{-1}(M^s_{m,n}(\mathbb{C}))$ where $M^s_{m,n}(\mathbb{C})$ is the set of the complex matrices with rank less then s, with s an integer number between 0 and $\min\{m, n\}$ such that $N < (m-s+2)(n-s+2)$, we will define the vanishing Euler characteristic of $(X, 0)$ and the Milnor number of a holomorphic function germ with an isolated singularity at X, $f : (X, 0) \to \mathbb{C}$.

Joint work with: *J. J. Nuño Ballesteros and J. N. Tomazella*

Global theorem of Quine for maps between surfaces

Catarina Mendes De Jesus (Universidade Federal de Viçosa)

Abstract. A map $f : M \longrightarrow N$ is said stable if exists a neighborhood V_f of f, in C^∞-Whitney topology, such that any map $g \in V_f$ is $\mathcal{A}$-equivalent to f (in the sense of diffeomorphism). The singular set Σf consists of curves of double points, possibly containing isolated cusp points. The branch set (i.e. the image of the singular set) consists of a number of immersed curves in the surface N (possibly with cusps) whose self-intersections are all transverse and disjoint from the cusps.

The non-singular set (which is immersed into the surface N by the map) consists of finitely many regions. Given orientations of the surfaces M and N, a region is positive if the map preserves orientation and negative otherwise. The singular set is the frontier of each half (positive or negative) of the surface M, i.e. any singular curve lies in the frontier of a positive and a negative region. We denote by M^+ (resp. M^-) the union of all

the positive (resp. negative) regions including their boundaries. Clearly, M^+ and M^- meet in their common boundary, the singular set of f.

An important result involving $M^\pm$ was demonstrated by Quine in "*A global theorem for singularities of maps between oriented 2-manifolds*, (1978)", applied by several authors, that seek invariants for the classification of stable maps.

Teorema (Quine): *Let f be a stable map between closed oriented surfaces M and N of degree $deg(f)$ then $\chi(M) - 2\chi(M^-) + C = deg(f)\chi(N)$, where χ denotes Euler characteristic and $C = C^+ - C^-$, the number of positive cusps minus the number of negative cusps.*

Ordinary algebraic curves

Daniel LEHMANN (Montpellier 2)

Abstract. In this paper, we describe the algebraic curves whose associated web is "ordinary"; their arithmetic genus is upper-bounded by a number stictly smaller than the Castelnuovo's bound. In particular, we describe the ordinary curves which are arithmetically Cohen Macaulay. Examples and counter-examples are given.

Joint work with: *Hantout and Gruson*

On the local geometry of definable stratified sets

David J. A. Trotman (University Of Provence)

Abstract. We analyse a definable function from a 1985 paper by W. Pawlucki. It is a counterexample to generalising his theorem concerning subanalytic sets with a smooth singular locus of codimension 1: such a set is Whitney (b)-regular if and only if it is locally a union of C^1 manifolds with boundary. Whitney (b) holds and the set is homeomorphic to a half-plane, but is not a C^1 manifold with boundary. It is also a counterexample to analogues of subanalytic results obtained by Orro and Trotman, does not satisfy Kuo's ratio test, and gives the first example of a definable stratification in an o-minimal structure which is (b)-regular but whose density is not continuous along strata. We give theorems indicating for which o-minimal structures definably stratified sets do satisfy similar properties to subanalytic sets, using earlier work of the second author.

Joint work with: *Guillaume Valette*

On the geometry of the cross-cap in the Minkoswki 3-space

Farid Tari (ICMC–USP)

Abstract. Parametrised surfaces in the Minkowski 3-space can have stable singularities of type cross-cap. We consider the local geometry of such surfaces at the cross-cap point. We start by obtaining special

parametrisations of the cross-cap. The induced metric can degenerate on a curve on the surface (labeled the locus of degeneracy (LD)). Also, on the Lorentzian part of the surface there may or may not be lines of principal curvature passing through each point. There is generically a curve labeled the Lightlike Principal Locus (LPL) that sperate the regions where there are two lines of principal curvature passing through each point and the region where there are none. We give the generic configurations of these special curves (LD, LPL, parabolic set) at the cross-cap. We also obtain the generic topological configurations of the lines of principal curvature at the cross-cap as well as those of the lightlike curves.

Joint work with: *Fabio Scalco Dias*

Isometric reduction of the codimension

Federico Sanchez Bringas (Facultad de Ciencias, Universidad Nacional Autonoma de Mexico)

Abstract. We study the problem of the existence of isometric immersions of submanifolds of a Riemannian n-manifold Q_c^n of constant sectional curvature c in totally geodesic submanifolds of lower dimensions, in such a way that the totality, or part of its principal configurations are preserved. We see how to ensure the existence of such immersions in terms of the existence of certain normal fields that we call Codazzi fields and analyze some properties of the curvature locus that imply their existence. We determine conditions implying the existence of an isometric immersion into a sphere. We analyze the existence of Codazzi fields in terms of the pairs (m, n), where m is the dimension of the submanifold and apply it to the study of the existence of second order non degenerate immersions proposed by Feldman.

Joint work with: *M.C. Romero Fuster*

One-parameter families of extrinsic differential geometries on hypersurfaces in hyperbolic space

Handan Yildirim (Istanbul University)

Abstract. In this talk, one-parameter families of extrinsic differential geometries on hypersurfaces in Hyperbolic space are constructed from a contact view point.

References:

1. V. I. Arnol'd, S. M. Gusein-Zade and A. N. Varchenko, *Singularities of Differentiable Maps, vol. I*, Birkhäuser, (1986).
2. M. Asayama, S. Izumiya, A. Tamaoki and H. Yıldırım, *Slant geometry of spacelike hypersurfaces in Hyperbolic space and de Sitter space*, Revista Matemática Iberoamericana, in press, (2012).
3. T. Banchoff, T. Gaffney and C. McCrory, *Cusps of Gauss mappings*, Research notes in Mathematics, Pitman, **55** (1982).

4. S. Izumiya, D-H. Pei and T. Sano, *Singularities of hyperbolic Gauss maps*, Proceedings of the London Mathematical Society, **86** (2003), 485-512.
5. S. Izumiya, D-H. Pei, M. C. Romero-Fuster and M. Takahashi, *Horospherical geometry of submanifolds in hyperbolic n-space*, Journal of London Mathematical Society, **71** (2005), 779-800.
6. S. Izumiya, D-H. Pei and M. C. Romero-Fuster, *The horospherical geometry of surfaces in Hyperbolic 4-space*, Israel Journal of Mathematics, **154** (2006), 361-379.
7. S. Izumiya and M. C. Romero Fuster, *The horospherical Gauss-Bonnet type theorem in hyperbolic space*, J. Math. Soc. Japan, **58** (2006), 965-984.
8. S. Izumiya, *Legendrian dualities and spacelike hypersurfaces in the lightcone*, Moscow Mathematical Journal, **9** (2009), 325-357.
9. S. Izumiya, *Horospherical geometry in the hyperbolic space*, Advanced Studies in Pure Mathematics, **55** (2009), 31-49.
10. S. Izumiya and H. Yıldırım, *Extensions of the mandala of Legendrian dualities for pseudo-spheres in Lorentz-Minkowski space*, Topology and its Applications, **159** (2012), 509-518.
11. S. Izumiya and H. Yıldırım, *Slant geometry of spacelike hypersurfaces in the lightcone*, J. Math. Soc. Japan, **63(3)** (2011), 715-752.
12. J. Martinet, *Singularities of Smooth Functions and Maps*, London Math. Soc. Lecture Note Series, Cambridge Univ. Press, **58** (1982).
13. J. A. Montaldi, *On contact between submanifolds*, Michigan Math. J., **33** (1986), 81-85.
14. G. Wasserman, *Stability of caustics*, Math. Ann., **210** (1975), 43-50.
15. V. M. Zakalyukin, *Lagrangian and Legendrian singularities*, Funct. Anal. Appl., **10** (1976), 23-31.
16. V. M. Zakalyukin, *Reconstructions of fronts and caustics depending one parameter and versality of mappings*, J. Sov. Math., **27** (1984), 2713-2735.

Joint work with: *Mikuri Asayama, Shyuichi Izumiya and Aiko Tamaoki*

Seifert manifolds as links of real singularities

HAYDEE AGUILAR-CABRERA (COLUMBIA UNIVERSITY)

Abstract. Given a real analytic d-regular function from $\mathbb{R}^n$ to $\mathbb{R}^p$, $(n \geq p)$, with an isolated singularity at the origin, a refinement of the Milnor fibration Theorem associates an open book decomposition of $\mathbb{S}^{n-1}$ to the singularity.

We present a family of real analytic d-regular functions f from $\mathbb{R}^4$ to $\mathbb{R}^2$ with isolated singularity at the origin. Let L_f be the link of the singularity of f. From f we define a function F from $\mathbb{R}^6$ to $\mathbb{R}^2$, with link L_F, such that F is d-regular, F has an isolated singularity at the origin and the knot $(\mathbb{S}^5, L_F)$ is a cyclic suspension of $(\mathbb{S}^3, L_f)$.

Then, for the family of functions F, we describe the topology of the link L_F and we show that in many cases the associated open books are different from those obtained from the Milnor fibrations of complex singularities.

Classification of metrically conical rational singularities

Helge Möller Pedersen (Aix-Marseille Université)

Abstract. Let $(X, 0)$ be a germ of complex singularity. Choose an embedding of $(X, 0)$ into $\mathbb{C}^N$ for some N. The hermitian metric of $\mathbb{C}^N$ induces a metric on X called the inner metric. The bilipschitz type of the inner metric is independent of the choice of embedding. It is well known that the topology of $(X, 0)$ is a cone over the link. A natural question is then when is the inner metric bilipschitz equivalent to a metric cone? We will give a complete answer to this question when $(X, 0)$ is a rational surface singularity, using the thick-thin decomposition of Birbrair, Neumann and Pichon.

Invariants of Topological Relative Right Equivalences

Imran Ahmed (ICMC-USP)

Abstract. The constancy of the Milnor number has several characterizations which were summarized by Greuel in 1986. This paper presents a study of these characterizations in the case of families of functions with isolated singularities defined on an analytic variety.

Joint work with: *Maria Aparecida Soares Ruas and João Nivaldo Tomazella.*

Fast and slow dynamics in nerve impulse

Isabel S Labouriau (Universidade do Porto)

Abstract. We will discuss the role of different time-scales in the dynamics of models for excitable tissue, like nerve impulse, and the associated geometry. These are important both in analysing existing models and in the construction of new ones.

On the total curvature of the real Amoeba

Jean-Jacques Risler (University Pierre et Marie Curie)

Abstract. A Caracterisation of Harnack Curves in a Toric Surface is given in terms of the total curvature of their real Amoeba. Some generalisation to Tropical geometry are given.

Joint work with: *Mikael Passare*

On contact round surgery

Jiro Adachi (Hokkaido University)

Abstract. A new construction method of contact manifolds is introduced. The construction is defined by using symplectic round handles. I would like to talk on how to define such things, and on some properties of the method.

Singularities of Axial Curvature Configurations on Surfaces in $\mathbb{R}^4$ and their Bifurcations

Jorge Sotomayor (IME–USP)

Abstract. The generic bifurcations of the singularities of the axial curvature configurations for one-parameter families of surfaces in $\mathbb{R}^4$, formed curves along which the normal curvature vector points in the direction of the axes of the Ellipse of Curvature, will be described.

Joint work with: *R. Garcia (UFG) and F. Spindola (CAPES - IME- USP)*

Generalizations of holomorphic surfaces in 4-space

Jose Basto-Goncalves (Universidade do Porto - Portugal)

Abstract. We consider some natural generalizations of surfaces conguent to graphs of holomorphic functions based on properties of their Gauss maps.

Topological triviality of families of map germs from $\mathbb{R}^2$ to $\mathbb{R}^2$

Juan Antonio Moya Pérez (Universitat de València)

Abstract. Abstract: We show that a 1-parameter unfolding $F : (\mathbb{R}^2 \times \mathbb{R}, 0) \to (\mathbb{R}^2 \times \mathbb{R}, 0)$ of a finitely determined map germ f is topologically trivial if it is excellent in the sense of Gaffney and the family of the discriminant curves $\Delta(f_t)$ is topologically trivial. We also give a formula to compute the number of cusps of 1-parameter unfoldings.

References:

1. T. Fukuda and G. Ishikawa, On the number of cusps of stable perturbations of a plane-to-plane singularity, *Tokyo J.Math.*, **10** (1987), 375–384.
2. T. Gaffney, Polar multiplicities and equisingularity of map germs, *Topology*, **32**, (1993), 185–223.
3. T. Gaffney, D. Mond, Cusps and double folds of germs of analytic maps $\mathbb{C}^2 \to \mathbb{C}^2$. *J. London Math. Soc.* (2) **43** (1991), no. 1, 185–192.
4. H.C. King, Topological type in families of map germs, *Invent. Math.*, **62** (1980), 1–13.
5. J.A.Moya-Pérez and J.J.Nuño-Ballesteros, The link of a finitely determined map germ from $\mathbb{R}^2$ to $\mathbb{R}^2$, *J. Math. Soc. Japan* 62 (2010), No. 4, 1069–1092.

6. J.A.Moya-Pérez and J.J.Nuño-Ballesteros, The link of a finitely determined map germ from $\mathbb{R}^2$ to $\mathbb{R}^2$, *to appear in Journals of Singularities.*
7. J.J.Nuño-Ballesteros, Topological triviality of families of map germs from $\mathbb{R}^2$ to $\mathbb{R}^3$, *Preprint*

Joint work with: *J.J Nuño Ballesteros*

Singularities of surfaces swept by constant curvature curves

Kentaro Saji (Kobe university)

Abstract. In this talk, we shall deal with surfaces swept by constant curvature curves in the constant curvature 3-spaces. Ruled surfaces and circular surfaces in $\mathbb{R}^3$ are examples of such surfaces. Since these surfaces have special properties and may have singularities, they are worth studying not only from the view point of classical differential geometry but also from the singularity theory. To construct such kind of surfaces, we use the "mandala and extended mandala" developed by Izumiya. We construct such surfaces and describe the conditions of generic singularities Moreover we shall point out a duality of conditions of singularities.

Joint work with: *Handan Yıldırım (Istanbul)*

Discriminants of quartics and quintics with Maple

León Kushner Schnur (Facultad De Ciencias, Unam-méxico)

Abstract. We study the discriminants of monic polynomials of degree 4 and 5. We relate their singularities with their zeroes. We also give moving picturs of these sets to the three dimensional space using Maple 15. Of course the parametrizations are also helped with the machine. We remark that this calculations will be extremely difficult without the machine, if not impossible.

Planar quasi-homogeneous singularities

Leonardo Meireles Câmara (Universidade Federal do Espírito Santo)

Abstract. In this talk we present some developments in the classification of germs of complex planar singularities. We give a complete set of invariants describing the moduli space of complex quasi-homogeneous curves. If possible, we describe the relationship between this classification and the one for quasi-homogeneous foliations and functions.

On generic surfaces in Anti de Sitter 3-space

Liang Chen (Northeast Normal University)

Abstract. We study the geometrical properties of general compact surfaces in Anti de Sitter 3-space from the view point of Legendrian singularity theory. We define the projective Gauss image on the compact surface and investigate the geometric meanings of singularities of this map. As applications we study the contact of general compact surfaces with some model surfaces.

Joint work with: *Shyuichi Izumiya and Masaki Kasedou*

On the transverse structure to the critical locus of a planar stable map

MAHITO KOBAYASHI (AKITA UNIVERSITY)

Abstract. We consider a graph transverse to the critical locus of a stable map of a manifold to the real plane, and study the restriction of the stable map to the inverse image of the graph. As application, we expose some arrangements of curves which are impossible to be critical loci, and also expose some explicit constructions of stable maps with prescribed critical loci.

Bounds for the multiplicity of the special fiber of blowups of modules.

MARCELA SILVA (UNIVERSIDADE ESTADUAL DE MARINGÁ - UEM)

Abstract. Let $(R, \mathfrak{m})$ be a Noetherian local ring and maximal ideal $\mathfrak{m}$ of dimension d, E a R-module of R^p with $\ell_R(R^p/E) < \infty$ and $\mathcal{F}(E)$ its special fiber of E. We use some classical invariants to give boundaries for the multiplicity f_0 of the special fiber of blowups of the symmetric algebra of R^p.

Singularities of spacecraft attitude control problems

MARK ROBERTS (UNIVERSITY OF SURREY)

Abstract. The problem of controlling the attitude of a spacecraft by a system of N control moment gyroscopes leads very naturally to that of lifting paths from angular momentum space R^3 to an actuator space isomorphic to the N-dimensional torus T^N via a map $h : T^N \to R^3$. The singularities of this map constitute significant obstacles to the design of optimal control algorithms. In this talk I will describe the control theory problems and show how they generate a number of interesting mathematical questions about the singularities and topology of the map h.

Asymptotic directions on spacelike surface of codimension 3 in de Sitter space

MASAKI KASEDOU (DEPARTMENT OF MATHEMATICS, HOKKAIDO UNIVERSITY)

Abstract. This talk is recent joint work with Maria Aparecida Soares Ruas (ICMC-USP) and Ana Claudia Nabarro (ICMC-USP). Asymptotic direction on the surface is defined as the kernel direction of

second fundamental form and it is good tool to study surface in Euclidean three space. Generically, singular point set of Gauss map consists of regular parabolic curves, which are classified into fold points and isolated cusp points. The asymptotic direction is transversal to the parabolic curve at fold points, and tangent to the parabolic curve at cusp points.

We will describe the case of spacelike surface in de Sitter five space. Second fundamental metric is obtained by choosing orthonormal sections on the spacelike surface. We consider one parameter family of normalized lightlike normals and introduce the notion of asymptotic direction on spacelike surface analogous to the case of the surface in Euclidean four space. We describe the relation between the asymptotic directions and second fundamental metric. There are up to four asymptotic directions at each point except for inflection and conic case on the spacelike surface.

References:

1. D.K. Hayashida Mochida, M.C. Romero Fuster, M.A. Soares Ruas, The Geometry of Surfaces in 4-Space from a Contact Viewpoint, Geom. Dedicata **54** (1995), 323-332.

Elimination of resonances in codimension one foliations by blow-ups

Miguel Fernandez Duque (University of Valladolid)

Abstract. The reduction of singularities of codimension 1 foliations in a three dimensional ambient space was obtained by F. Cano in the article *Reduction of the singularities of codimension one singular foliations in dimension three*. In that work, the final singularities obtained generalize the ones described by Seidenberg in the two dimensional case. The process is divided in two steps: first one gets pre-simple singularities; afterwords the final simple singularities may be obtained from the pre-simples ones by eliminating resonances using blow-ups. In the present work we show how one can eliminate the resonances of pre-simple singularities in any dimensional ambient space, using blow-ups which are centered in non singular subvarieties of the ambient space, in order to get simple singularities. In this way we would complete the second stage of a hypotethical reduction of singularities in any dimension, which so far is an open problem.

Sphere eversion and their plane images (new method)

Minoru Yamamoto (Aichi University of Education)

Abstract. An oriented spheare in 3-space is eversible under a regular homotopy (continuous deformation of immersion). S. Smale first proved in 1958 that it is possible to evert a sphere, Since then several visualizations are given by Phillips, Morin, Thurston and Francis. In this talk, by using a theory of deforming stable maps into the plane, we describe a new way of eversion where the behavior of apparent counter is quite simple. We also visualize how the curves of self-intersection are deformed.

Joint work with: *Mikami Hirasawa (Nagoya Institute of Technology)*

The Wigner caustic on shell and singularities of odd functions

PEDRO DE M RIOS (ICMC–USP)

Abstract. We study singularities of the Wigner caustic on shell, for Lagrangian submanifolds of affine symplectic spaces. We present the physical background and motivation for studying the Wigner caustic on shell and present its mathematical definition in terms of a generating family. Because generating families of Wigner caustics on shell must be deformations of odd functions, we study simple singularities in the category of odd functions and their odd versal deformations, applying these results to classify singularities of Wigner caustics on shell. We also interpret these singularities in terms of the local geometry of the Lagrangian submanifolds.

Joint work with: *Wojciech Domitrz and Miriam Manoel*

Affine equidistants and their evolution

PETER GIBLIN (UNIVERSITY OF LIVERPOOL)

Abstract. Affine equidistants in 3-space are defined by points of a surface in $\mathbf{R}^3$ at which tangent planes are parallel. We consider the locus of points on the parallel tangent chords joining these pairs which are a fixed ratio of the distance from one end. Thus the ratio one-half is special and results from the collapse of two equidistants onto one another. The locus of singularities of equidistants is the 'centre symmetry set' which is also the envelope of parallel tangent chords.

The geometry and singularity theory of these equidistants and their evolution will be described; it has several highly unusual features.

Joint work with: *Vladimir Zakalyukin and Paul Warder*

On the geometry of singular surfaces

RAÚL OSET SINHA (ICMC-USP)

Abstract. We study the flat geometry of singular surfaces in $\mathbb{R}^3$. These singular surfaces can occur as projections of smooth surfaces M in $\mathbb{R}^4$ to $\mathbb{R}^3$. We recover some aspect of the geometry of M from those of its singular projections..

Joint work with: *Farid Tari*

Stratified transversality of holomorphic maps

SAURABH TRIVEDI (AIX-MARSEILLE UNIVERSITÉ)

Abstract. We give a characterization of Whitney a-regular complex analytic stratifications in a complex manifold N in terms of the topology on the set of holomorphic maps with N as the target. Our result can be seen as a complex analogue of Trotman's theorem.

Singular flat webs and Frobenius manifolds

Sergey Agafonov (UFPB)

Abstract. The theory of Frobenius manifolds, having its origin in theoretical physics, has deep interrelations with apparently very different areas of mathematics: Witten-Gromov invariants and quantum cohomology, deformation of flat connections, integrable systems, singularity etc. We discuss a new aspect of this fruitful and fast developing theory: its relations with the classical chapter of differential geometry, namely the web theory. Using the structure of a given semi-simple Frobenius 3-fold, we construct a 3-web in the plane. This web enjoys the following properties:

1) it is flat,

2) it admits at least one-dimensional symmetry algebra and

3) its Chern connection remains holomorphic in singular points, where at least 2 web directions coincide.

We present a classification of singularities of 3- webs with such properties and show that any such web is obtained by the presented construction. We give also a geometrical interpretation of the associativity equation, describing the corresponding Frobenius 3-fold.

Topology of the set of singular values of stable maps between surfaces

T. Yamamoto (Kyushu Sangyo University)

Abstract. Let $f_0\colon M \to N$ be a smooth map between manifolds. In this talk, for a given series "A" of countable singularities of a stable map $M\ to\ N$, the notion "A-minimal contour" of f_0 is introduced. We investigate some A-minimal contour of a smooth map between surfaces.

Whitney umbrellas and swallowtails

Takashi Nishimura (Yokohama National University)

Abstract. We introduce map germs of pedal unfolding type and the notion of normalized Legendrian map-germs. We show that the fundamental theorem of calculus provides a natural one to one correspondence between Whitney umbrellas of pedal unfolding type and normalized swallowtails.

Affine differential geometry of plane curves

Takashi Sano (Hokkai-Gakuen University)

Abstract. We study the affine invariants of plane curves from the view point of the singularity theory of smooth functions. We also study them from a different angle.

The warping degree of a nanoword

Tomonori Fukunaga (Hokkaido University)

Abstract. A. Kawauchi has introduced the notion of warping degrees of knot diagrams and A. Shimizu has given an inequality for warping degrees and crossing numbers of knot diagrams . In this talk, we extend the notion of warping degrees and Shimizu's inequality to nanowords. Moreover, to describe the condition for the equality, we introduce the new notion on nanowords, "the alternating nanowards", which corresponds to the alternating knot diagrams.

Fronts of Whiney umbrella

Toshizumi Fukui (Saitama University)

Abstract. We describe the differential geometric ingredients for Whitney umbrella, which is known as the only stable singularity of a surface to 3-dimensional Euclidean space. We present several criteria of the singularity types of fronts of Whitney umbrella in terms of these differential geometric language.

Joint work with: *Masaru Hasegawa*

The exponential map at a cuspidal singularity

Vincent Grandjean (UFC Fortaleza)

Abstract. The aim of this talk is to describe the local properties of the exponential map at the singular point of a cusp singular surface.

The exponential map is well defined at any smooth point of a Riemannian manifold. It allows a local parameterization of a neighbourhood of the considered point called "normal coordinates", which is the Riemannian avatar of the plane polar coordinates and 3-space spherical coordinates. These coordinates are very useful to estimate for instance the volume of a ball of small radius. A (metrically) conic point is a point of a singular manifold, which locally is a cone over its link and equipped with a conic metric. It turns out that smooth points of Riemmanian manifold are "sleeping conic points": Melrose and Wunsch proved (2004) the existence of normal coordinates studying an exponential mapping at conic points of a singular Riemannian manifolds. They use the frame of conic manifolds, that is a manifold with compact smooth boundary equipped with a conic metric along the boundary to prove their result - then they are set-up to study the wave equation on conic singular spaces or equivalently on conic manifolds after blowing-up of the conic point.

As a matter of fact, the study of the local Riemannian geometry in a neighbourhoood of singularity of a singular manifold - think of an algebraic set embedded in an Euclidean space and whose regular part is equipped with the restriction of the ambient metric - is unknown. As we will see in the talk, even though it may mostly have been due to a lack of work on such a topic, once out of the conic or conic-like singularities things can become wild, even for surfaces.

Cusp singularities are more metrically degenerate than conic singularities, but any small neighbourhood of the singularity is still a topological cone over its link. As for a conic point we work with a cusp manifold, introduced by us, which is a manifold with compact smooth boundary equipped with a cusp metric along the boundary. Cheeger horn metrics are "very special cases" of cusp-metrics. As we will see the dynamics of the geodesics reaching a cusp singular point of a cusp surface can be very different from that of a conic point.

Joint work with: *D. Grieser (Universitat Oldengburg)*

Lipschitz geometry of complex singularities in higher dimensions.

Walter D Neumann (Barnard College, Columbia University)

Abstract. The talk will be an overview of work in which many people are involved (Birbrair, Fernandes, Grandjean, O'Shea, Pichon and others) to understand the local Lipschitz geometry of complex varieties in higher dimensions. It will include results, examples and speculations.

Reaching generalized critical values of a polynomial

Zbigniew Jelonek (Polish Academy of Sciences)

Abstract. Let $f : \mathbb{K}^n \to \mathbb{K}$ be a polynomial, $\mathbb{K} = \mathbb{R}, \mathbb{C}$. We give an algorithm to compute the set of generalized critical values. The algorithm uses a finite dimensional space of rational arcs along which we can reach all generalized critical values of f.

Joint work with: *Krzysztof Kurdyka*

4.4. **Poster Sessions.**

Stable maps of surfaces in the 2-sphere

ALANA CAVALCANTE FELIPPE (UNIVERSIDADE FEDERAL DE VIÇOSA)

Abstract. Let $f : M \longrightarrow N$ be a stable map, where M is a closed surface. According to Whitney, [1] the singular set of f is made of a set of simple curves (without intersections), closed and disjoint in M. These curves separate the surface M into connected components. Meanwhile, the image of the singular set (apparent contour) is a set of curves in N with a finite number of transverse intersections and isolated cusp points. In [2] the authors associated graphs with weights at the vertices to these stable maps that codify the topological type of their singular and regular sets. In this work we determine which are the graphs that can be associated to stable maps from surfaces into the sphere. They also find a necessary and sufficient condition that tells us when some graph can be the graph of a fold map.

References:

1. M. Golubitsky and V. Guillemin, *Stable Mappings and Their Singularities, Springer Verlag, Berlin (1976).*
2. D. Hacon, C. Mendes de Jesus and M.C. Romero Fuster, Graphs of stable maps from closed orientable surfaces to the 2-sphere, *Journal of Singularities,* 2 67–80, 2010.

Joint work with: *C. Mendes de Jesus*

An algorithm to construct the matrix of a presentation and applications

ALDICIO JOSÉ MIRANDA (UNIVERSIDADE FEDERAL DE ALFENAS)

Abstract. Let $(\mathcal{X}, x) \in \mathcal{O}_\ell$ be a multi-germ of a n-dimensional Cohen-Macaulay variety and $\mathcal{O}_{(\mathcal{X},x)}$ denotes the set of germs g in $\mathcal{O}_\ell$ such that $g(\mathcal{X}) = 0$. For a fixed finite analytic map germ $f : (\mathcal{X}, x) \to (\mathbb{C}^{n+1}, 0)$ we have that $\mathcal{O}_{(\mathcal{X},x)}$ is a finite $\mathcal{O}_{n+1}$-module via the function f^*. If the classes of $g_1, g_2, \ldots, g_h$ in $\frac{\mathcal{O}_{(\mathcal{X},x)}}{f^*\mathfrak{m}_0}$ generate it as vector space over $\mathbb{C}$, then $g_1, g_2, \ldots, g_h$ generate $\mathcal{O}_{(\mathcal{X},x)}$ as $\mathcal{O}_{n+1}$-module. A presentation of $\mathcal{O}_{(\mathcal{X},x)}$ over $\mathcal{O}_{n+1}$ is an exact sequence

$$\mathcal{O}^h_{n+1} \xrightarrow{\lambda} \mathcal{O}^h_{n+1} \xrightarrow{\alpha} \mathcal{O}_{(\mathcal{X},x)} \longrightarrow 0 \tag{1}$$

of $\mathcal{O}_{n+1}$-modules. Mond and Pellikaan presents an algorithm to obtain the matrix λ. See [1] for more details. We modify the algorithm given by Mond-Pellikan and we have implemented it in Maple and Singular Softwares to calculate such matrix λ. You can use this implementation and Fitting ideals to obtain the k^{th}-multiple points set (counting multiplicity), of the application f.

References:

1. Mond, D. and Pellikaan, R., *Fitting ideals and multiple points of analytic mappings.* Algebraic geometry and complex analysis (Pátzcuaro, 1987), 107–161, Lecture Notes in Math., 1414, Springer, Berlin, 1989.

Joint work with: *Guillermo Peñafort-Sanchis and Marcelo Escudeiro Hernandes*

Global phase portraits of a SIS model

Alex Carlucci Rezende (ICMC–USP)

Abstract. In the qualitative theory of ordinary differential equations, we can find many papers whose objective is the classification of all the possible topological phase portraits of a given family of differential system. Most of the studies rely on systems with real parameters and the study consists of outlining their phase portraits by finding out some conditions on the parameters. Here, we studied a susceptible-infected-susceptible (SIS) model described by the differential system $\dot{x} = -bxy - mx + cy + mk$, $\dot{y} = bxy - (m + c)y$, where b, c, k, m are real parameters with $bm \neq 0$. Such system describes an infectious disease from which infected people recover with immunity against reinfection. The integrability of such system has already been studied by Nucci and Leach (2004) and Llibre and Valls (2008). We found out two different topological classes of phase portraits.

Joint work with: *Regilene D. S. Oliveira*

Unfoldings of real functions

Alex Paulo Francisco (UNESP-Campus de São José do Rio Preto)

Abstract. Let $F : \mathbb{R} \times \mathbb{R}^r, (t_0, x_0) \to \mathbb{R}$ be a smooth function. We can naturally regard F as an r-parameter family of functions $F_x : \mathbb{R}, t_0 \to \mathbb{R}$ or as a 1-parameter family of functions $F_t : \mathbb{R}^r, x_0 \to \mathbb{R}$. We are interested in r-parameter families of functions, which is called an unfolding of F_{x_0}. The existence of versal unfoldings of real functions is one of central results. We study three criteria for versality seeking recognize versal unfoldings. An interesting result is that given the dimension r of the parameter space and the type of singularity A_k of the unfolded function, the bifurcation set and the singular set are unique, up to diffeomorphism.

As applications of unfolding's theory, we study some very concrete geometrical problems involving distance-squared and height functions.

References:

1. Bruce, J. W. and Giblin, P. J. *Curves and Singularities: a geometrical introduction to singularity theory.* 2^{nd}. ed. Cambridge University Press, 1992.

An extension of hamiltonian systems to the thermodynamic phase space

Asahi Tsuchida (Hokkaido University)

Abstract. Contact geometry is a suitable framework in order to deal with irreversible thermodynamical processes. I will introduce a paper about geometrodynamics written by D.Eberard,B.M.Maschke and A.J.van der schaft. And I will mainly explain about conservative contact systems.

On Monge characteristic systems of hyperbolic differential systems

ATSUSHI YANO (HOKKAIDO UNIVERSITY)

Abstract. We consider the relations between Monge characteritic systems $\mathcal{M}_i$ of hyperbolic and parabolic differential system (R, D) and Monge characteristic systems $\hat{\mathcal{M}}_i$ of the prolongation of (R, D). Here, the hyperbolic (or parabolic, elliptic) differential system (R, D) is a pair of a smooth manifold R and a subbundle D of the tangent bundle over R that satisfy certain conditions, and can be regarded as a geometric formulation of a certain class of PDE systems, which contains single second order PDEs $F(x, y, z, z_x, z_y, z_{xx}, z_{xy}, z_{yy}) = 0$ and determined systems $G(x, y, z, w, z_x, z_y, w_x, w_y) = H(x, y, z, w, z_x, z_y, w_x, w_y) = 0$. Generally, for hyperbolic and parabolic differential systems, we can define Monge characteristic systems $\mathcal{M}$ of the systems, which are rank 2 differential systems on R. The differential system $(\hat{R}, \hat{D})$ obtained from a hyperbolic (or parabolic, elliptic) differential system through prolongation is of the same type, and hence we can consider the Monge characteristic systems $\hat{\mathcal{M}}_i$ of $(\hat{R}, \hat{D})$. Then, we investigate relations between $\mathcal{M}_i$ and $\hat{\mathcal{M}}_i$.

On the Morse inequalities

BRENO RAFAEL PINHEIRO SAMPAIO (UNIVERSIDADE FEDERAL DO CEARÁ)

Abstract. This work is to stimulate the study of singularities, where such a feat to make use of the Morse inequalities, which is an important tool of information between a variety of topology and critical points of a real-valued function on it. Where such a study for Morse produced a series of inequalities, that for some undergraduate students are a real mystery to be understood, which often diverts the focus of this audience to other areas of mathematics.

The geometry of the focal surface and of the focal curve

CARLA WOLF (UNIVERSIDADE FEDERAL DE SANTA MARIA)

Abstract. The focal surface of a curve γ in the Euclidean 3-space, is defined as the envelope of the normal planes of γ. The focal surface of γ is singular along a curve C_γ, called the *focal curve* or *generalized evolute*. This curve is given by the centers of the osculating spheres of γ. In this work we study the geometry of the focal surface, focusing on the properties of the focal curve. These concepts can be generalized for curves in $\mathbb{R}^{m+1}$. The focal curve may be parametrized in terms of the Frenet frame of γ, $\{\mathbf{t}, \mathbf{n}_1, \cdots, \mathbf{n}_m\}$, as $C_\gamma(t) = (\gamma + c_1\mathbf{n}_1 + ... + c_m\mathbf{n}_m)(t)$. The coefficients $c_1, \cdots, c_m$ are smooth functions called *focal curvatures* of γ. It is then obtained a formula relating the Euclidean curvatures of γ with its focal curvatures. Defining

a vertex of a curve in $\mathbb{R}^n$ as a point at which the curve has at least $(n+2)$-point contact with its osculating hypersphere, we give necessary and sufficient conditions for a point of γ to be a vertex. In such points the focal surface of a space curve γ is locally diffeomorphic to the swallowtail surface. This work is part of a master dissertation in mathematics and was based on R. Uribe-Vargas [2] and M. C. Romero Fuster [1].

References:

1. M. C. Romero-Fuster and E. Sanabria-Codesal, *Generalized evolutes, vertices and conformal invariants of curves in* $\mathbb{R}^{n+1}$. Indagationes Mathematicae, **10**:2 (1999), 297-305.
2. R. Uribe-Vargas, *On vertices, focal curvatures and differential geometry of space curves.* Bull. of Brazilian Math. Soc., **36**:3 (2005)285-307.

Joint work with: *Claudia Candida Pansonato*

First order local Vassliev type invariants of stable maps of oriented 3-manifolds in $\mathbb{R}^4$

Catiana Casonatto (Universidade Federal de Uberlândia)

Abstract. In this work we obtain that the space of first order local Vassiliev type invariants of stable maps of oriented 3-manifolds in $\mathbb{R}^4$ is 4-dimensional. We give a geometric interpretation for two of the four generators of this space, namely, I_Q the number of quadruple points and $I_C = P$ the number of pairs of points of crosscap/plane type, of the image of a stable map.

Joint work with: *Roberta Godoi Wik Atique and María del Carmen Romero Fuster*

Generic bifurcation of refracted systems

Claudio A Buzzi (Ibilce - Unesp - São José Do Rio Preto - Sp.)

Abstract. In this work we study systems with refraction. More precisely, let $K \subseteq \mathbb{R}^3$ be a compact set and $\Sigma \subseteq K$ be given by $\Sigma = f^{-1}(0)$, where f is a smooth function $f : K \longrightarrow \mathbb{R}$ having $0 \in \mathbb{R}$ as a regular value (i.e. $\nabla f(p) \neq 0$, for any $p \in f^{-1}(0)$). Clearly Σ is the separating boundary of the regions $\Sigma_+ = \{q \in K | f(q) \geq 0\}$ and $\Sigma_- = \{q \in K | f(q) \leq 0\}$. Consider the space of vector fields $Z : K \longrightarrow \mathbb{R}^3$ such that

$$Z(x,y,z) = \begin{cases} X(x,y,z), & \text{for} \quad (x,y,z) \in \Sigma_+, \\ Y(x,y,z), & \text{for} \quad (x,y,z) \in \Sigma_-, \end{cases} \tag{2}$$

where X and Y are smooth vector fields defined on K. We are interested in the study of discontinuous systems having the property $Xf(p) = Yf(p)$ for all p in Σ. These systems are known as refracted systems. These systems turn out to be relevant for applications in relay systems. In our approach we follow the Smale's Program to this classe of systems, i.e, we classify the generic singularities of codimensions zero and one.

Joint work with: *J. C. Medrado and M. A. Teixeira*

Indefinite improper affine spheres and indefinite affine minimal surfaces with singularities

DAISUKE NAKAJO (KYUSHU UNIVERSITY)

Abstract. We introduce representation formulae for indefinite improper affine spheres and indefinite affine minimal surface (possibly with singularities). Using these formulae, we study several properties of these surfaces.

TBA

ELIRIS CRISTINA RIZZIOLLI (IGCE–UNESP, RIO CLARO)

Abstract. TBA

Some results on pairs of involutions

ELIZABETH SALAZAR FLORES RUTH (ICMC–USP)

Abstract. In this work we present the study of pairs (φ_1, φ_2) of involutions (φ_1, φ_2) on $(\mathbb{R}^n, 0)$ associated with divergent diagrams of folds (f_1, f_2). We also obtain normal forms for the pairs (φ_1, φ_2) and, consequently, normal forms for the corresponding divergent diagram of folds, when the involutions are linear and transversal. In addition, we present a discussion of our results to the study of a class of discrete reversible dynamical systems. Finally, we give a characterization of the group generated by any pair of linear involutions.

On the motion under focal attraction in a rotating medium: a generalization

FRANCISCO DE MELO VIRÍSSIMO (IME–USP)

Abstract. In his classical book, H.K. Wilson [1] proposes the following system of ODE's:

$$x' = -\omega y + v\frac{R-x}{\sqrt{(R-x)^2+y^2}}$$

$$y' = \omega x - v\frac{y}{\sqrt{(R-x)^2+y^2}}$$

According to Wilson, this set of ODE's decribes a model for the motion of entities (such as particles or micro-organisms) with scalar velocity equal $v > 0$, under a focal attraction (by the focus $(R, 0)$). Here, $\omega \geq 0$ is the angular velocity of the medium. This problem was solved by Sotomayor [2,3] for the $\mathbb{R}^2$ case on the disk $D = \{(x, y) | x^2 + y^2 = R^2\}$ of radius $R > 0$. In this work, we will discuss this model for the case where

the domain is the cylinder and the non-trivial case on the sphere.

References:

1. WILSON, H.K.. *Ordinary Differential Equations*; Addison-Wesley (1971).
2. SOTOMAYOR, Jorge. *Lições de Equações Diferenciais*; Projeto Euclides, IMPA-CNPq (1979).
3. SOTOMAYOR, Jorge. *On the motion under focal attraction in a rotating medium*; Bull. Belg. Math. Soc. - Simon Stevin 15, pags. 921-925 (2008).

Configurations of Principal Curvature Lines on Piecewise Smooth Surfaces

Gláucia Aparecida Soares Miranda (IME–USP)

Abstract. The main goal of this work is to study the principal configuration of a piecewise smooth surface, $S = S^+ \cup C \cup S^- \subset I\!R^3$, in a neighborhood of the origin, analyzing the cases in which the principal curvature lines (of the surfaces S^+ and S^-) have quadratic contact or cross transversely the boundary C. For our purposes, consider that along the curve C, the surface S is C^0.

By using the "Regularization Method" along the boundary C, we will see that, under certain conditions, from a point on the curve C of quadratic contact, bifurcate only Darbouxian umbilic points of types D_1 and D_3 on the regularized surface S_ε ($\varepsilon > 0$ small). In the case in which the point on C is of transversal crossing we obtain, on S_ε, only the D_3 type.

References:

1. R. Garcia and J. Sotomayor. Umbilic and Tangential Singularities on Configurations of Principal Curvature Lines. *Anais da Academia Brasileira de Ciências*, vol.74, n.1, p.1-17 (2002).
2. R. Garcia and J. Sotomayor. *Differential Equations of Classical Geometry, a Qualitative Theory.* 27° Colóquio Brasileiro de Matemática, IMPA, Rio de Janeiro, Brasil (2009).
3. C. Gutierrez and J. Sotomayor. Structurally Stable Configurations of Lines of Principal Curvature. *Asterisque*, 98-99, p.195-215 (1982).

Symmetric functions and applications to multiple point spaces in corank 2

Guillermo Peñafort Sanchis (Universidade Federal de Valência)

Abstract. Given a map $f : (\mathbb{C}^n, 0) \to (\mathbb{C}^p, 0), n < p$, we give explicit expressions for the generators of its lifted double point space, $D^2(f)$, with is formed by the pairs $(z, z') \in \mathbb{C}^{2n}$ such that $f(z) = f(z')$ and, if $z = z'$, then f is singular in z. $>$ We also give a model for its quotient, $D^2(f)/S_2$, given by the identification of pairs (z, z') and (z', z).

Geometric singularities for solutions of single conservation laws

Hitoshi Kurokawa (Hokkaido University)

Abstract. In this poster I introduce the paper "Geometric Singularities for Solutions of Single Conservation Laws" written by Shyuichi Izumiya and Georgios T. Kossioris. Single conservation laws play an important role in various fields, e.g., gas dynamics and oil reservoir problems. Now I state the geometric framework for study of the generation and propagation of shock wave appearing in weak solutions of their.

Reversible-equivariant Belitskii normal form

Iris de Oliveira (ICMC–USP)

Abstract. In a dynamical system, the presence of symmetries or reversing symmetries leads to the occurrence of multiple solutions: symmetries take trajectories in trajectories, preserving the direction with time, whereas reversing symmetries take trajectories in trajectories, reversing direction with time. When both occur simultaneously, the system is called reversible-equivariant and all these objects have structure of group, the group of symmetries and reversing symmetries of the system, denoted by Γ. This fact implies the existence of a normal subgroup of index 2, formed by symmetries of Γ and denoted by Γ_+. Thus, the mathematical formulation for this theory is made through the group representation theory. The study begins by considering a group homomorphism

$$\sigma : \Gamma \to \mathbf{Z}_2, \tag{3}$$

where $\mathbf{Z}_2$ is the multiplicative group $\{-1, 1\}$ e $\Gamma_+ = \ker(\sigma)$. A vector field $g : V \to V$ is called Γ-reversible-equivariant if

$$g(\gamma x) = \sigma(\gamma)\gamma g(x), \ \forall \gamma \in \Gamma, \ x \in V. \tag{4}$$

In the study of many dynamical systems, a method of great interest is to pass the vector field for the normal form Belitskii. In fact, this has been a very efficient tool for the study of bifurcations, the occurrence of periodic solutions, limit cycles and other local and global phenomena. In this work, we deduce how to obtain this normal form preserving all symmetries and reversing symmetries of system. The method is based on the representation theory of algebraic groups and invariant theory.

Jacobian Conjecture: a different approach

Jean Venato Santos (UFU)

Abstract. Let $F : \mathbb{R}^n \to \mathbb{R}^n$ be a polynomial local diffeomorphism and let S_F denote the set of not proper points of F. The Jelonek's real Jacobian Conjecture states that if $codim(S_F) \geq 2$, then F is bijective. We present a weak version of such conjecture establishing the sufficiency of a necessary condition for bijectivity. Furthermore, we will discuss how our result contributes for the Jacobian Conjecture.

Topological and bi-Lipschitz contact equivalence of map germs

João Carlos Ferreira Costa (Ibilce/Unesp)

Abstract. A central question in Singularity theory is the local classification of mappings up to diffeomorphisms. However, this is a difficult problem and it presents a lot of rigidity. Then it seems natural to investigate the classification of mappings given by equivalence relations in which the change of coordinates are weaker than diffeomorphisms. In this work we present some advances in the study of the topological and bi-Lipschitz contact equivalence of map germs. These equivalence relations are weaker versions of classical contact equivalence (or $\mathcal{K}$-equivalence) introduced by John Mather.

Two Morse functions and singularities of the product map

Kazuto Takao (Osaka University)

Abstract. It is known that any two Morse functions on a closed manifold can be related by a homotopy consisting of Morse functions except for finitely many births and deaths of canceling pairs of critical points. Few things however are known about the minimum number of births and deaths. We give an upper bound for this number in terms of the number of cusp points of the product map of the two Morse functions.

Characterization of germs of irreducible plane curves with maximal torsion

Leandro Nery de Oliveira (Universidade Federal do Acre)

Abstract. This paper aims to summarize the main results obtained in Master's Thesis "Caracterização dos Germes de Curvas Planas Irredutíveis com Torção Maximal", defended in 2011 in the Master's Program in Mathematics, Federal University of Espírito Santo. Let $f \in \mathbb{C}[[X,Y]]$ a formal series convergent in a neighborhood of the origin $\mathbb{C}^2$ and irreducible, we define the equation $f(X,Y) = 0$ (also indicated as (f) or C) as the *germ of an analytic curve* or an *irreducible plane algebroid curve*. We can find a ring associated to f, namely, $\mathcal{O} = \frac{\mathbb{C}[[X,Y]]}{\langle f \rangle} = [[x,y]]$; where x and y are the classes of X and Y in $\mathcal{O}$ and $\langle f \rangle$ is the ideal generated by f. It is known that $\mathcal{O}$ is a local ring, ie, has a single maximal ideal. Is possible to calculate $\mathcal{O}d\mathcal{O}$ defined as the $\mathcal{O}$-module of Kahler differentials and we can define the $\mathcal{O}$-submodule torsion $\mathcal{T}$ of $\mathcal{O}d\mathcal{O}$ as $\mathcal{T} = \{\omega \in \mathcal{O}d\mathcal{O} : \xi\omega = 0, \text{ for some } \xi \in \mathcal{M} \setminus \{0\}\}$, with $\mathcal{M}$ the maximal ideal of $\mathcal{O}$. The problem of classification of germs of analytic plane curves has been studied by many mathematicians since the pioneering work of Isaac Newton in the XVII century. He introduced the notions of what is now called Newton polygon and the Newton-Puiseux expansion. There are works in this direction also in the XIX century, which stand out Kronecker and M. Noether. The classification of germs of plane curves, to the topological type, was carried out between 1920 and 1930 mainly by Brauner, Burau, Kahler and Zariski. In 1963, Berger published the article *Differentialmoduln Eindimensionaler Lokaler Ringe*, proving that the length of $\mathcal{O}$-submodule torsion $l(\mathcal{T})$ is equal to $l(\overline{\mathcal{O}}d\overline{\mathcal{O}}/\mathcal{O}d\mathcal{O}) + l\left(\overline{\mathcal{O}}/\mathcal{O}\right)$, where $\overline{\mathcal{O}}$ is the integral closure of $\mathcal{O}$ and $\overline{\mathcal{O}}d\overline{\mathcal{O}}$ is the $\mathcal{O}$-module of Kahler differentials of $\overline{\mathcal{O}}$. In his article *Characterization of Planes Algebroid*

Curves Whose Module of Differentials has Maximum Torsion, 1966, Zariski used this result to help establish a criterion for a germ of irreducible curve to be (analytically) quasi-homogeneous, ie, when the length of submodule torsion of Kahler differentials is equal to the length of the algebraic conductor of local ring on its algebraic closure, that is, when $l(T) = c$ where $c = l(\mathfrak{C})$, being $\mathfrak{C}$ is the algebraic conductor of the closure integral of $\mathcal{O}$, or $\mathfrak{C} = \{g \in \overline{\mathcal{O}} : \nu_\varphi(g) \geq c\} = t^c\overline{\mathcal{O}} \subset \mathcal{O}$ where ν_φ is the function value and t is a uniformizing parameter (ie, $\nu_\varphi(t) = 1$). Therefore we are interested in the length of the torsion submodule $\mathcal{T}$ of $\mathcal{O}d\mathcal{O}$ and to know the characteristic of the curve C when the length is maximum. An interesting result is that relates all the curve C, which may be represented by a Puiseux parameterization $x = t^n$ e $y = t^m + \sum_{i>m} a_i t^m$, with a curve which has a short representation C', ie, given by $x = t^n$ e $y = t^m + \sum_{i>m}^{q} a_i' t^{v_i}$, com $v_i \notin \nu_\varphi(\mathcal{O} \setminus \{0\})$. From this result, we conclude that a necessary and sufficient condition for a curve C to be quasi-homogeneous is when the length of the torsion submodule $\mathcal{T}$ of $\mathcal{O}d\mathcal{O}$ is maximum.

References:

1. Berger, R. *Differentialmoduln Eindimensionaler Lokaler Ringe.* Math. Zeitschr. 81, 326–354 1963.
2. Hefez, A. *Irreducible Plane Curve Singularities.* In Real and Complex Singularities, D. Mond and M. J. Saia, Editors, Lecture Notes in Pure and Applied Math. Vol. 232, Marcel Dekker, 1-120, 2003.
3. Hefez, A. & Hernandes, M. *The Analytic Classification of Plane Branches.* Bulletin of the London Mathematical Society 2011; doi: 10.1112/blms/bdq113.
4. Kunz, E. *Introduction to Plane Algebraic Curves.* Birkhäuser Boston, 2005.
5. Oliveira, L. N. *Caracterização dos Germes de Curvas Planas Irredutíveis com Torção Maximal.* Dissertação de Mestrado: UFES, 2011.
6. Zariski, O. *Characterization of Plane Algebroid Curves whose Module of Differentials has Maximum Torsion.* Proc. Nat. Acad. of Sc. 56: 781-786, 1966.

A model of dissipation in the N-body problem

Lucas Ruiz dos Santos (Universidade de São Paulo)

Abstract. A variation of the Newtonian N-body problem is purposed in this work. Dissipative forces are included in this setting and the long-time behaviour of the solutions is analysed. It is noted that the bounded solutions which are far of the collisions have periodic orbits as their $\omega-$limit sets, and all of them are contained in the same plane, for a given initial condition.

Joint work with: *Clodoaldo Grotta Ragazzo*

On the periodic solutions of a class of Duffing differential equations

Luci Roberto (IBILCE - UNESP)

Abstract. In this work we study the periodic solutions, their stability and bifurcation for the class of Duffing differential equation $x'' + cx' + a(t)x + b(t)x^3 = \lambda h(t)$, where $c > 0$ is a constant, λ is a real parameter,

$a(t)$, $b(t)$ and $h(t)$ are continuous T–periodic functions. Our results are proved using the averaging method of first order.

Joint work with: *Jaume Llibre (UAB)*

Polar of irreducibles curves with semigroup $\langle 2p, 2q, 2pq + d\rangle$

Mauro Fernando Hernandez Iglesias (UFF)

Abstract. We will prove that, the generic polar of a generic curve with semigrup $\langle 2p, 2q, 2pq + d\rangle$, is a Newton nodegenerete curve, therefore its irreducibles components are of genus less than two, also see that the topology of the polar does not depend of d, that always has a component with characteristic pair (p, q), and the remaining components are obtained from the partial fraction decomposition of $\frac{q}{p}$.

CR submanifolds

Mirjana Milijevic (Hokkaido University)

Abstract. We generalize some results that are valid on real hypersurfaces to CR submanifolds of maximal CR dimension.

Affine space curves and homogeneous surfaces

Na Hu (Hokkaido University)

Abstract. We study the affine space curves and give the classification of the affine space curves with constant curvatures. As the affine space curves with constant curvatures can be seen as the homogeneous curves, we can find the one parameter affine groups of the curves and then we compare the relations between these groups with the ones of the affine homogeneous surfaces.

Classification at infinity of polynomials of degree 3 in 3 variables

Nilva Rodrigues Ribeiro (Universidade Federal de Viçosa/ Campus Rio Paranaíba)

Abstract. The aim of this work is to classify polynomials of degree 3 in three variables, $f(x_0, x_1, x_2) = f_2(x_0, x_1, x_2) + f_3(x_0, x_1, x_2)$, where f_i is homogeneous of degree i for $i = 1.2$. The results will appear in [3]. The article by Bruce Wall and [2] gives a description of the type of singularity at infinity of f, where f_2 is a node, binode or unode. We refine this classification, giving a more complete description of the influence of the singularities of f_3 in the classification of the singularities at infinity of f.
Based on Siersma and Tibar's results in [1], we also study the topology of the generic fibre at infinity.

References:

1. D. Siersma, M. Tibar, *Betti Bounds of Polynomials*, math. AG 2011.
2. J.W Bruce and C. T. C Wall, *On the classification of cubic surfaces*, J.london Math. Soc(2) 19 (1979), 245-256.
3. Classification at infinity of polynomials of degree 3 in 3 variables, preprint.

Envelopes and apparent contour of surfaces

Pedro Benedini Riul (UNESP - Câmpus de São José do Rio Preto)

Abstract. We study the envelope, or the discriminant, of a smooth family $F : \mathbb{R}\times\mathbb{R}^r \to \mathbb{R}$. In particular, for $r = 2$, we study the relation between the envelope and the geometry of the apparent contour of a regular surface $M = F^{-1}(0)$, where 0 is a regular value of F.

We are also interested in the local structure of envelopes. Let $\Sigma \subset \mathbb{R} \times \mathbb{R}^r$ be the set constituted by critical points of the projection $\pi : M \to \mathbb{R}^r$ given by $\pi(t, x) = x$. An interesting result is that the envelope of F is the projection $\pi(\Sigma)$, being Σ, under one condition, locally a parametrized $r - 1$ manifold in $\mathbb{R} \times \mathbb{R}^r$ as well as $\pi(\Sigma)$ in $\mathbb{R}^r$.

We also study examples in which we find regular points and points of regression of envelopes.

References:

1. Bruce, J. W. and Giblin, P. J. *Curves and Singularities: a geometrical introduction to singularity theory*. 2^{nd}. ed. Cambridge University Press, 1992.

The effect of singular perturbation at typical singularities of Filippov systems

Pedro Toniol Cardin (UNESP - Ilha Solteira)

Abstract. The object of this work is the study of systems of ordinary differential equations having the form

$$\dot{x} = \begin{cases} F(x,y,\epsilon) & \text{if} \quad h(x,y,\epsilon) \leq 0, \\ G(x,y,\epsilon) & \text{if} \quad h(x,y,\epsilon) \geq 0, \end{cases} \qquad \epsilon\dot{y} = H(x,y,\epsilon),$$

where $\epsilon \in \mathbb{R}$ is a small parameter, $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ and $y \in \mathbb{R}$ denote the slow and fast variables, respectively, and $F = (F_1, \ldots, F_n)$, $G = (G_1, \ldots, G_n)$, h and H are C^r functions.

Our main question is to understand as typical singularities of Filippov systems are affected by singular perturbation. We extend the Geometric Singular Perturbation Theory obtained in [1] to these systems.

References:

1. Fenichel, N. (1979). Geometric singular perturbation theory for ordinary differential equations, *J. Diff. Equations* **31**, 53–98.

Joint work with: *Paulo Ricardo da Silva and Marco Antonio Teixeira*

Polar Multiplicities and Euler Obstruction for Ruled Surfaces

Rodrigo Martins (Universidade Estadual de Maringá)

Abstract. Given two integers $n \geq m \geq 0$ we exhibited (ruled) surfaces with multiplicity n and Euler obstruction m.

Joint work with: *Nivaldo G. Grulha Jr. and Marcelo E. Hernandes*

Homologia semialgébrica sobre corpos reais fechados

Rodrigo Mendes Pereira (Universidade Federal do Ceará)

Abstract. Esta dissertação está baseada em uma serie de trabalhos publicados por H. Delfs e M. Knebusch sobre uma teoria de homologia para espaços semialgébricos sobre corpos reais fechados. Neste trabalho, reunimos as definições e principais resultados sobre a teoria de homologia semialgébricas. Além disso, como aplicação dessa teoria, trazemos uma prova do Teorema de Ax-Grothendick para aplicações polinomiais sobre corpos reais fechados.

Curves on a spacelike surface in Lorentz-Minkowski 3-space

Takami Sato (Hokkaido University)

Abstract. In this poster, we consider curves on a spacelike surface in Lorentz-Minkowski 3-space. We introduce new geometric invariants for these curves. As an application of the unfolding theory of functions, we give a classification of singularities of associated mappings of our curves for understanding the local geometric propertes of these invariants. We also investigate global properties of these invariants.

The Cusp-Fold singularity on $\mathbb{R}^3$ (bifurcations of non-smooth vector fields on $\mathbb{R}^3$)

Tiago de Carvalho Carvalho (UNESP-Bauru)

Abstract. This paper is concerned with the local bifurcation analysis around typical codimension one singularities of piecewise smooth dynamical systems in $\mathbb{R}^3$. Generic one−parameter families of a class of non−smooth vector fields presenting the so called Cusp−Fold singularity are studied and the bifurcation diagrams are exhibited. In such bifurcation diagrams we observe the occurrence of T−singularities. In our main results the structural stability and the asymptotical stability of the Cusp−Fold singularity is discussed.

Joint work with: *Marco Antonio Teixeira*

Generic properties of singular trajectories of control affine system

Wataru Yukuno (Hokkaido University)

Abstract. I would like to introduce the proprieties of the singular trajectories of the generic affine control system. The critical point of the end-point mapping defined on the set of admissible controls is called singular control. The absolutely continuous solution with respect to its singular control is called singular trajectory. The admissible control is singular if and only if there exists a lift of singular trajectory which is called adjoint vector such that it satisfies the constrained Hamiltonian equation. By using this equivalent properties, I'll introduce the proprieties of the singular trajectories of the generic affine control system.

Lightcone dualities for curves in the sphere

Yang Jiang (Hokkaido University)

Abstract. In this talk we consider the curves in the unit 2-sphere and unit 3-sphere. The unit sphere can be canonically embedded in the lightcone and de Sitter space in Minkowski space. We investigate these curves in the framework of the theory of Legendrian dualities between pseudo-spheres in Minkowski space.

Joint work with: *Shyuichi Izumiya and Donghe Pei*

Webs on sphere by pencils of planes

Yasuhiro Kurokawa (Shibaura Institute of Technology)

Abstract. We will discuss about webs on sphere made by being cut off by pencils of planes in $\mathbb{R}^3$.

A symplectic framework for gravitational lensing

Yoshiki Izumikawa (Hokkaido University)

Abstract. Light is curved by strong gravity and this phenomenon acts as a lens sometimes - gravitational lensing. It can be discribed in the framework of symplectic geometry. In this poster, I introduce S.Izumiya and S.Janeczko's work "A symplectic framework for multiplane gravitational lensing".

Constructing generators for the module of liftable vector fields

Yusuke Mizota (Graduate School of Mathematics, Kyushu University)

Abstract. The notion of liftable vector fields was introduced by Arnol'd for studying bifurcations of wave front singularities. Liftable vector fields is defined for a multigerm $f : (K^n, S) \to (K^p, 0)$ and have various applications. The module of vector field liftable over f is denoted by L_f and when $n < p$, to the best of author's knowledge, there seems to have been few general results. Nishimura gave, in principle, a method to construct generators for L_f when $n \leq p$ in some conditions. However, it is difficult to obtain

generators by hand by his method. We give a method to find polynomial liftable vector fields and obtain explicit generators by a computer.

5. List of Participants

1. Abdulrahman Faleh A Aljohani / Liverpool University
2. Abramo Hefez / Universidade Federal Fluminense
3. Ageu Barbosa Freire / Universidade Federal da Paraíba
4. Alana Cavalcante Felippe / Universidade Federal de Viçosa
5. Aldicio José Miranda / Universidade Federal de Alfenas
6. Alex Carlucci Rezende / ICMC–USP
7. Alex Paulo Francisco / UNESP-Campus de São José do Rio Preto
8. Alexander Fernandes Fonseca / USP
9. Alexandre Fernandes / Universidade Federal do Ceará
10. Alisson de Carvalho Reinol / UNESP - Presidente Prudente
11. Ana Claudia Nabarro / ICMC-USP-São Carlos
12. Anderson Maciel / Universidade Federal de Santa Maria
13. Andras Szucs / Eotvos University
14. André Vanderlinde da Silva / USP
15. Andrea Sacramento / ICMC–USP
16. Anne Pichon / Aix-Marseille Université
17. Antonio Carlos Nogueira / Faculdade de Matemática - Universidade Federal de Uberlândia
18. Antonio Andrade do Espirito Santo / ICMC–USP
19. Aryampilly Jayanthan Parameswaran / Tata Institute of Fundamental Research
20. Asahi Tsuchida / Hokkaido University
21. Atsushi Yano / Hokkaido University
22. Aurelio Menegon Neto / UNAM
23. Breno Rafael Pinheiro Sampaio / Universidade Federal do Ceará
24. Bruna Oréfice Okamoto / Universidade Federal de São Carlos
25. Bruno de Paula Jacóia / Universidade de São Paulo
26. Carla Wolf / Universidade Federal de Santa Maria
27. Carles Bivià-Ausina / Universitat Politècnica de València
28. Carlos Alberto da Silva Junior / Universidade Federal de São João del Rei - UFSJ
29. Carolina Fernandes Molina Sanches / Universidade Federal de São João del Rei
30. Catarina Mendes De Jesus / Universidade Federal de Viçosa
31. Catiana Casonatto / Universidade Federal de Uberlândia
32. Claudio A Buzzi / Ibilce - Unesp - São José do Rio Preto - Sp.
33. Clementa Alonso / University of Alicante
34. Christophe Eyral / ICMC–USP
35. Daisuke Nakajo / Kyushu University
36. Daniel Lehmann / Montpellier 2
37. Darwin Emerson Guerrero Vejarano / Universidade Estadual Paulista

38. David Mond / University of Warwick
39. David J. A. Trotman / University of Provence
40. Debora Lopes da Silva / Universidade de São Paulo
41. Edison Righetto / Unesp/ Ilha Solteira
42. Edison Marcavillaca Niño de Guzmán / Pontificia Universidad Católica del Perú
43. Eduardo Alfonso Chincaro Egusquiza / UFMG
44. Eliris Cristina Rizziolli / IGCE - Unesp/Rio Claro
45. Elizabeth Salazar Flores Ruth / ICMC–USP
46. Emilio Carvalho Carvalho / Universidade Estadual de Maringá
47. Érica Boizan Batista / Instituto de Biociências Letras e Ciências Exatas - UNESP
48. Fabio Scalco Dias / Universidade Federal de Itajubá
49. Farid Tari / ICMC–USP
50. Fausto Assunção Lira / USP
51. Federico Sanchez Bringas / Facultad de Ciencias, Universidad Nacional Autonoma de Mexico
52. Flávio Rocha / IMPA
53. Francisco de Melo Viríssimo / Universidade de São Paulo
54. Francisco Vieira Oliveira / Universidade Federal da Paraiba
55. Gláucia Miranda / USP
56. Goo Ishikawa / Hokkaido University
57. Grazielle F. Barbosa / Universidade Federal de São Carlos
58. Guillermo Peñafort Sanchis / Universitat de Valencia
59. Handan Yildirim / Istanbul University
60. Hans Schonemann / Universität Kaiserslautern
61. Haydee Aguilar-Cabrera / Columbia University
62. He Dong Pei / Northeast Normal University
63. Helge Möller Pedersen / Aix-Marseille Université
64. Hitoshi Kurokawa / Hokkaido University
65. Imran Ahmed / ICMC–USP
66. Ingrid Sofia Meza Sarmiento / ICMC–USP
67. Iris de Oliveira Zeli / ICMC–USP
68. Isabel S Labouriau / Universidade do Porto
69. Jackson Itikawa / ICMC–USP
70. Javier Fernandez de Bobadilla / CSIC
71. Jawad Snoussi / Universidad Nacional Autônoma de México
72. Jean Cerqueira Berni / USP
73. Jean Venato Santos / UFU
74. Jean-Jacques RISLER / University Pierre et Marie Curie
75. Jesus Enrique Achire / UNICAMP
76. Jiro Adachi / Hokkaido University

77. João Carlos Ferreira Costa / Ibilce/Unesp
78. Jorge Alberto Coripaco Huarcaya / USP
79. Jorge Sotomayor / USP
80. Jorge Luiz Deolindo Silva / ICMC-USP
81. Jorge Vitório Pereira / IMPA
82. Jose Seade Kuri / Instituto de Matematicas, Universidad Nacional Autonoma de Mexico
83. Jose Basto-Goncalves / Universidade do Porto - Portugal
84. Juan José Nuño-Ballesteros / Universitat de Valencia
85. Juan Antonio Moya Pérez / Universitat de València
86. Julie Lapébie / Université d'Aix-Marseille (AMU)
87. Kazuto Takao / Osaka University
88. Kentaro Saji / Kobe university
89. Kevin Houston / University of Leeds
90. Kotaro Yamada / Tokyo Institute of Technology
91. Laís da Silva Oliveira / Universidade de São Paulo
92. Lê Dung Trang / ICTP-Italy
93. Leandro Nery de Oliveira / Universidade Federal do Acre
94. León Kushner Schnur / Facultad De Ciencias, Unam-méxico
95. Leonardo Meireles Meireles Câmara / Universidade Federal do Espírito Santo
96. Liang Chen / Northeast Normal University
97. Lizandro Sanchez Challapa / Universidade Federal da Paraíba
98. Lucas Ruiz dos Santos / Universidade de São Paulo
99. Luci Roberto / IBILCE - UNESP -SJRP
100. Luciana De Fátima Martins / UNESP - Campus de São José do Rio Preto
101. Luis Fernando Mello / Universidade Federal de Itajubá
102. Luis Florial Espinoza Sánchez / ICMC-USP
103. Mahito Kobayashi / Akita University
104. Marcela Silva / Universidade Estadual de Maringá - UEM
105. Marcelo Buosi / Universidade Federal dos Vales do Jequitinhonha e Mucuri
106. Marcelo José Saia / Universidade de São Paulo
107. Maria Amélia de Pinho Barbosa Hohlenwerger / ICMC–USP
108. Maria Aparecida Soares Ruas / ICMC-USP
109. Maria Carmen Romero Fuster / Universitat de València
110. Mark Roberts / University of Surrey
111. Masaaki Umehara / Tokyo Institute of Technology
112. Masaki Kasedo / Department of Mathematics, Hokkaido University
113. Masatomo Takahashi / Muroran Institute of Technology
114. Mauro Fernando Hernández Iglesias / UFF
115. Michelle F. Z. Morgado / UNESP - IBILCE

116. Miguel Fernandez Duque / University of Valladolid
117. Minoru Yamamoto / Aichi University of Education
118. Miriam Garcia Manoel / Universidade de São Paulo
119. Miriam S. Pereira / UFPB
120. Mirjana Milijevic / Hokkaido University
121. Mostafa Salarinoghabi / ICMC
122. Na Hu / Hokkaido University
123. Napoleón Caro Tuesta / Universidade Federal da Paraíba
124. Nayene Michele Pitta / Universidade Estadual de Maringá
125. Nicolas Dutertre / Aix-Marseille Université
126. Nilva Rodrigues Ribeiro / Universidade Federal de Viçosa/ Campus Rio ParanaÍba
127. Patrícia Hernandes Baptistelli / Universidade Estadual de Maringá
128. Patricia Tempesta / Universidade Estadual de Maringá
129. Patrick Popescu-Pampu / Lille 1
130. Pedro de M Rios / ICMC–USP
131. Pedro Benedini Riul / Universidade Estadual Paulista - UNESP - Câmpus de São José do Rio Preto
132. Pedro Toniol Cardin / UNESP - Ilha Solteira
133. Pedro Henrique Apoliano Albuquerque Lima Lima / Universidade de São Paulo
134. Peter Giblin / University of Liverpool
135. Raimundo N. Araújo dos Santos / Universidade de São Paulo - São Carlos
136. Raúl Oset Sinha / ICMC-USP
137. Regilene D S Oliveira / ICMC-USP
138. Roberta Godoi Wik Atique / USP
139. Roberto Callejas-Bedregal / Universidade Federal da Paraíba-UFPB
140. Rodrigo Mendes Pereira / Universidade Federal do Ceará
141. Rodrigo Martins / Universidade Estadual de Maringá
142. Saurabh Trivedi / Aix-Marseille Université
143. Sergey Agafonov / UFPB
144. Shihoko Ishii / University of Tokyo
145. Shuichi Izumiya / Hokkaido University
146. Stanislaw Janeczko / Institute of Mathematics, Polish Academy of Sciences
147. Takahiro Yamamoto / Kyushu Sangyo University
148. Takami Sato / Hokkaido University
149. Takashi Nishimura / Yokohama National University
150. Takashi Sano / Hokkai-Gakuen University
151. Terence J Gaffney / Northeastern University
152. Thiago Henrique Freitas / Universidade de São Paulo
153. Tiago de Carvalho Carvalho / UNESP-Bauru
154. Tomonori Fukunaga / Hokkaido University

155. Ton Marar / ICMC–USP
156. Toru Ohomoto / Dept. Math., Hokkaido University
157. Toshizumi Fukui / Saitama University
158. Valmecir Bayer Bayer / UFES - Vitória - ES
159. Vanessa Munhoz Reina Bezerra / Universidade Federal da Grande Dourados
160. Victor Hugo Jorge Pérez / Universidade de Sao Paulo-USP-São Carlos
161. Vincent Grandjean / UFC Fortaleza
162. Walter D Neumann / Barnard College, Columbia University
163. Wataru Yukuno / Hokkaido University
164. Yang Jiang / Hokkaido University
165. Yasuhiro Kurokawa / Shibaura Institute of Technology
166. Yoshiki Izumikawa / Hokkaido University
167. Yusuke Mizota / Graduate School of Mathematics, Kyushu University
168. Zbigniew Tadeusz Jelonek / Polish Academy of Sciences