Book of abstracts
The *Workshops on Real and Complex Singularities* form a series of biennial meetings organized by the Singularities group at Instituto de Ciências Matemáticas e de Computação of São Paulo University (ICMC-USP), Brazil. Their main purpose is to bring together world experts and young researchers in singularity theory, applications and related fields to report recent achievements and exchange ideas, addressing trends of research in a stimulating environment.

The thirteenth edition, to be held from the 27th of July to the 1st of August of 2014, is celebrating the 60th birthday of professor María del Carmen Romero Fuster. This time, after the workshop, a School on Singularity theory and Real and Complex Singularity Days of Young Researchers have also been organized, to be held from the 4th to the 8th of August. The present book is the collection of abstracts of the mini-courses, plenary talks, parallel sessions, short talks and posters that shall be presented during these two weeks.

The Workshop poster was created by Guillermo Peñafort-Sanchis.

**Contents**
1. 13th International Workshop on Real and Complex Singularities

1.1. Plenary Talks.

**Flexes, flex equivalences and versal unfoldings**

Andrew du Plessis (Aarhus University)

Recent work on flexes of curves by Dias, Nuño-Ballesteros, Oset Sinha, Tari and Wall has given rise to satisfactory notions of equivalence for germs of curves which also take into account tangential singularities e.g. in the sense of points of inflexion. These forms of equivalence do not, however, seem to lead to a satisfactory unfolding theory. I will propose an alternative, flex equivalence, which does yield an unfolding theory, complete with versal and stable unfoldings; and I will discuss its relationship with the approaches mentioned above.

**Equisingularity of families of Isolated Determinantal Singularities**

Bruna Orêfice Okamoto (Universidade Federal de São Carlos)

We study necessary and sufficient conditions for a family of isolated determinantal singularities to be Whitney or topologically equisingular. The topological triviality of the family is related to the constancy of the vanishing Euler characteristic and the Whitney equisingularity is characterized in terms of the constancy of the polar multiplicities. We generalize the results of Lê-Ramanujam and Teissier for hypersurfaces and the results of Gaffney for complete intersections.

Joint work with J.J. Nuño-Ballesteros and J.N. Tomazella

**On subarrangements of the affine Weyl arrangements**

Hiroaki Terao (Hokkaido University)

A Weyl arrangement is the arrangement defined by the root system of a finite Weyl group. It is composed of the reflecting hyperplanes of all the reflections in the Weyl group. The Weyl arrangement is a free arrangement (or free divisor or Saito divisor) and has been studied in many contexts including the singularity theory. Each Weyl arrangement defines the affine Weyl arrangement whose members are all the translated hyperplanes by integers. Thus each affine Weyl arrangement has infinitely many affine hyperplanes. We recently found many free subarrangements of affine Weyl arrangements. We also constructed explicit bases for their logarithmic derivation modules. In my talk, I will discuss various subarrangements of affine Weyl arrangements. I am especially interested in the Shi, Catalan, Shi-Catalan, ideal, ideal-Shi arrangements. The works I will mention in the talk include joint works with Takuro Abe, Mohamed Barakat, Michael Cuntz, Torsten Hoge, Daisuke Suyama among others.
Dicriticos y valores atípicos de polinomios
Ignacio Luengo (Universidad Complutense de Madrid)
TBA

Two time-scales in coupled FitzHugh-Nagumo equations
Isabel Labouriau (Universidade do Porto, Portugal)
We discuss the role of different time-scales in the dynamics of two coupled FitzHugh-Nagumo equations, and the associated geometry. When two different time-scales are considered separately, the dynamics is constrained by the geometry of the slow manifold, that in this specific example is a surface in the 4-dimensional phase space. Interesting dynamical behaviour arises at singularities of the projection of this surface into the plane of slow variables. Even more interesting is the situation when the slow equation has a zero at a fold point.

Lecture on the work of Carmen Romero Fuster
Juan José Nuño-Ballesteros (Valencia), R. Oset-Sinha (UFSCar) and M. A. S. Ruas (ICMC)

Forbidden values for the total Milnor number of projective hypersurfaces
June Huh (University of Michigan Ann Arbor)
Which values are possible as the total Milnor number of a degree d projective hypersurface with isolated singularities? I will explain how to assign a base point when applying the Lefschetz hyperplane theorem, and use this technique to give an affirmative answer to a conjecture of Dimca and Papadima on the total Milnor number. A more general conjecture on the total Milnor number will be proposed.

On the Arnold’s conjecture on hyperbolic homogeneous polynomials
Federico Sánchez Bringas (Universidad Nacional Autonoma de México)
A polynomial $f$ is called hyperbolic if its graph is a surface with only hyperbolic points off the origin. The set $\text{Hyp}(n)$ constituted by homogeneous hyperbolic polynomials of degree $n$ is a topological subspace of $\mathbb{R}[x, y]$. The connectedness of this space has been studied as part of the subject known as the Hessian Topology introduced in [2], [4], and named by V. I. Arnold in [1] and [3] (problems 2000-1, 2000-2, 2001-1, 2002-1). This property of the space depends on the degree of the polynomials that constitute it. That is, $\text{Hyp}(3)$ and $\text{Hyp}(4)$ are connected subspaces meanwhile $\text{Hyp}(6)$ is a disconnected one. According to this, V.I. Arnold stated the following conjecture [3], p.139: “The number of connected components of the space of hyperbolic homogeneous polynomials of degree $n$ increases as $n$ increases (at least as a linear function of $n$).”
We present a proof of this conjecture. Our approach uses index properties at singularities of hyperbolic quadratic differential forms.

Joint work with Adriana Ortíz Rodríguez.

References


Mumford Theorem In Lipschitz Geometry

Lev Birbrair (Universidade Federal do Ceará)

The classical Theorem of Mumford states that a topologically regular Complex Algebraic Surface in \( \mathbb{C}^3 \) with an isolated singular point is smooth. We proof that any Lipschitz Regular Complex Algebraic set is smooth. No restriction on the dimension is needed. No restriction of Singularity to be isolated is needed.

Joint work with Le Dung Trang, Alexandre Fernandes and Edson Sampaio.

Milnor fibration for mixed functions with non-isolated singularities

Mutsuo Oka (Tokyo University of Science)

Convenient mixed functions with strongly non-degenerate Newton boundary has a Milnor fibration. This is proved in my paper M. Oka. "Non-degenerate mixed functions" Kodai Math. J., 33(1):1–62, 2010. Convenience implies that the singularity is isolated. In this paper, we consider strongly non-degenerate mixed functions without the convenience assumption. The dimension of the singularities can be bigger. We first gives a Lojasiewicz type inequality and then we prove the existence of the Milnor fibration for such mixed functions.

Singular fibers and visualization of multivariate data

Osamu Saeki (Kyushu University)

this talk, we first classify singular fibers of stable maps of compact 3-manifolds with boundary into surfaces. Then, we give two applications. One is a construction of a certain cobordism invariant for functions on surfaces with boundary. As the second application, we use the singular fibers
to visualize three dimensional multivariate data, by combining a recent technique in computer science, called the joint contour net.

Local symmetries and geometry of curves and surfaces
Peter Giblin (Liverpool)
Local symmetries of curves and surfaces and their connexion with geometry have been investigated by many authors, and various 'symmetry sets' have arisen as a result, often related to the 'flat' or 'spherical' geometry of the manifold. I shall survey some of these ideas, and present some recent work with S. Janeczko on local symmetry of surfaces in 3-space via reflexion through a point, which turns out to be related to earlier work on 'centre symmetry sets'.

How to construct simple multigerms
Roberta Wik Atique (ICMC-USP)
A natural question is: how to combine monogerms (branches) in order to obtain simple multigerms? Mather has shown that a stable multigerm is made of stable branches and their analytic strata have regular intersection. In a paper with D. Mond and T. Cooper we show that multigerms of codimension 1 are made of stable branches and their analytic strata have almost regular intersection. Suppose we have a simple s-germ and add for example either an immersion or a fold, do we obtain a simple (s + 1)-germ? In this talk we will describe the operations of augmentation and concatenations in order to obtain multigerms and discuss the simplicity.

Joint work with Maria Aparecida Ruas and Raul Oset Sinha.

The Witten deformation for singular spaces and radial Morse functions
Ursula Ludwig (Paris, Orsay)
The Witten deformation is an analytical method proposed by Witten in the 80’s which, given a Morse function \( f : M \to \mathbb{R} \) on a smooth compact Riemannian manifold \( M \), leads to a proof of the famous Morse inequalities.

The aim of this talk is to present a generalisation of the Witten deformation to a singular space \( X \) with cone-like singularities and radial Morse functions. As a result one gets Morse inequalities for the \( L^2 \)-cohomology, or dually for the intersection homology of the singular space \( X \). Moreover, as in the smooth theory, one can relate the Witten complex, \( i.e. \) the complex generated by the eigenforms to small eigenvalues of the Witten Laplacian, to an appropriate geometric complex (a singular analogue of the smooth Morse-Thom-Smale complex).

Radial Morse functions are inspired from the notion of a radial vectorfield on a singular space. Radial vectorfields have first been used by Marie-Hélène Schwartz to define characteristic classes on singular varieties.
The topology of wave fronts and caustics
Vyacheslav Sedykh (Gubkin University of Oil and Gas, Moscow)

We will start with problems of the Contact Geometry of space curves. These problems are related to the Global Singularity Theory of wave fronts. We will speak on the adjacencies of simple multisingularities of stable Legendre mappings. The local and global properties of Lagrange mappings will be considered as well.

Even Dimensional Improper Affine Spheres
Wojciech Domitriz (Warsaw University of Technology)

There are two different types of bi-dimensional improper affine spheres. The non-convex ones can be modeled by the center-chord transform of a pair of planar curves while the convex ones can be modeled by a holomorphic map. Both constructions can be generalized to arbitrary even dimensions: the former class corresponds to the center-chord transform of a pair of Lagrangian submanifolds while the latter is related to special Kähler manifolds.

In this paper we show that the improper affine spheres obtained in this way are solutions of certain exterior differential systems. We also discuss the stability of singularities of these improper affine spheres from the Lagrangian and the Legendrian points of view.

Joint work with Marcos Craizer and Pedro de M. Rios.

The Thom Conjecture for proper polynomial mappings
Zbigniew Jelonek (Institute of Mathematics - Polish Academy of Science)

Let $f, g : X \to Y$ be continuous mappings. We say that $f$ is topologically equivalent to $g$ if there exist homeomorphisms $\Phi : X \to X$ and $\Psi : Y \to Y$ such that $\Psi \circ f \circ \Phi = g$. Let $X, Y$ be complex smooth irreducible affine varieties. We show that every algebraic family $F : M \times X \ni (m, x) \mapsto F(m, x) = f_m(x) \in Y$ of polynomial mappings contains only a finite number of topologically non-equivalent proper mappings. In particular there are only a finite number of topologically non-equivalent proper polynomial mappings $f : \mathbb{C}^n \to \mathbb{C}^m$ of bounded (algebraic) degree. This gives a positive answer to the Thom Conjecture in the case of proper polynomial mappings.
1.2. Mini-course.
A walk in the world of perverse sheaves

Jean-Paul Brasselet (CNRS and Aix-Marseille University)

Aim of the course will be to "demystify" the objects called "perverse sheaves", to show their usefulness and (some of) their applications.

Lecture 1: The first part of the walk will be "perverse". I will start slowly, like in a walk in the mountains, providing motivations, with the local calculus for intersection homology. I will provide examples and useful properties, letting time to enjoy the landscape.

Lecture 2: The second stage of the walk will be change of the landscape, going through sheaf theory. I will discuss basic results on sheaves and cohomology of sheaves. I will provide various definitions of perverse sheaves and their properties, still with examples.

Lecture 3: During the third part of the walk, I will discuss applications of perverse sheaves. According to the time (and the weather), I will take participants through the valleys of de Rham theorem, decomposition theorem, Morse theory, Lefschetz theorems, nearby and vanishing cycles (cf Lectures by David Massey the following week).

1.3. Parallel Sessions.

The geometry of quadratic vector fields with irreducible invariant hyperbolas

Alex Carlucci Rezende (ICMC-USP)

In this talk we consider the class $QS_f$ of all quadratic systems possessing a finite number of singularities (finite and infinite ones). A quadratic polynomial differential system can be identified with a single point of $\mathbb{R}^{12}$ through its coefficients. Herein, using the algebraic invariant theory, we provided necessary and sufficient conditions for a system in $QS_f$ to have irreducible invariant hyperbolas in terms of its coefficients. We also considered the number and multiplicity of such irreducible hyperbolas. We give here the global bifurcation diagram of the class $QS_f$ of systems with invariant hyperbolas. The bifurcation diagram is done in the 12-dimensional space of parameters and it is expressed in terms of polynomial invariants. The results can therefore be applied for any family of quadratic systems in this class, given in any normal form.

Joint work with Regilene D. S. Oliveira and Nicolae Vulpe.
Curves on a timelike surface
Andrea de Jesus Sacramento (ICMC-USP)

Our aim is introduce the notion of pseudo-spherical evolutes of curves on a timelike surface in three dimensional Minkowski space. We investigate the singularities and geometric properties of pseudo-spherical evolutes such curves. Furthermore, we investigate the relation of the de Sitter (hyperbolic) evolute of a spacelike curve in $S^2_1$ with the lightlike surface along the curve in $\mathbb{R}^3$.

The epsilon multiplicity and polar varieties.
Antoni K Rangachev (Northeastern University)

The epsilon multiplicity is a generalization of the Buchsbaum-Rim multiplicity for submodules of free modules not necessarily of finite colength. In this talk building on the theory of pairs of modules and the multiplicity polar theorem introduced by Gaffney, we relate the epsilon multiplicity to the geometry of certain polar varieties of modules. We discuss applications of this relation to Thom-Whitney equisingularity theory for families of arbitrary isolated singularities.

Singularity, controllability and observability of port-Hamiltonian systems
Asahi Tsuchida (Hokkaido University)

A port-Hamiltonian system is a kind of generalized Hamiltonian system defined on Dirac manifolds and plays a important role on many-body systems and so on. From the viewpoint of control theory, we see some relations among controllability, observability and singularity of the port-Hamiltonian systems.

Fold singularities in spacelike CMC surfaces
Atsufumi Honda (Miyakonojo National College of Technology)

Fold singularity on maximal surfaces (i.e. spacelike zero mean curvature surfaces) plays an important role. For example, it is known that if a maximal surface admits fols singular points, it can be extended uniquely to a type-changing zero mean curvature surface. In this talk, we will show the non-existence of fold singularities on spacelike CMC surfaces and discuss some related topics.

Joint work with M. Koiso.

The geometric genus of hypersurface singularities
Baldur Sigurðsson (Central European University)
Using the path lattice cohomology we provide a conceptual topological characterization of the geometric genus for certain complex normal surface singularities with rational homology sphere links, which is uniformly valid for all superisolated and Newton nondegenerate hypersurface singularities. In this talk we will focus on the Newton nondegenerate case.

Joint work with A. Nemethi.

**Rotation Numbers for Planar Attractors of Equivariant Homeomorphisms**

Begoña Alarcón Cotillas (Universidade Federal Fluminense)

Given an integer $n > 1$ we consider $\mathbb{Z}_n$-equivariant and orientation preserving homeomorphisms of the plane with an asymptotically stable fixed point at the origin. We present examples without periodic points and having some complicated dynamical features. The key is a preliminary construction of $\mathbb{Z}_n$-equivariant Denjoy maps of the circle.

**A Darboux type theorem for germs of holomorphic one dimensional foliations**

Bruno Scárdua (IMPA)

We show that a germ of a holomorphic one dimensional foliation at a singularity in a space of dimension two admits a holomorphic first integral if, and only if, there are infinitely many closed leaves, a finite number of separatrices and each separatrix has linearizable holonomy. Indeed, if there are infinitely many closed leaves and the set of separatrices is finite then either the foliation admits a holomorphic first integral or it admits a formal simple integrating factor of Darboux type.

Joint work with César Camacho.

**Multiplicity and Łojasiewicz exponent of generic linear sections of monomial ideals**

Carles Bivià-Ausina (Universitat Politècnica de València)

We show an inequality that relates the multiplicity and the sequence of Łojasiewicz exponents of generic linear sections, or mixed Łojasiewicz exponents, of a given ideal of $\mathcal{O}_n$. We analyze the monomial case and, as a consequence we obtain an interesting class of monomial ideals that can be characterized combinatorially. We also present some inequalities that relate mixed multiplicities and log canonical thresholds of ideals of arbitrary codimension.

**Infinitesimal Hartman-Grobman theorem in dimension three**

Clementa Alonso González (University of Alicante)
In this talk we give the main ideas to show that a real analytic vector field in $\mathbb{R}^3$ with a singular point at the origin is locally topologically equivalent to its principal part defined through Newton polyhedra under non-degeneracy conditions.

**Gauss Maps and Duality of Sphere Bundles**

Daniel Dreibleib (University of North Florida)

Given an immersion of a hypersurface into Euclidean space, it has been well established that there is a duality between the set of degenerate height functions and the set of degenerate projections into hyperplanes. In previous work, the author has shown that this result can be generalized to a duality between the singularity sets of sphere bundles and their Gauss maps. However, the unfolding structure of the height functions/projections allows for stronger results regarding the duality. In this talk, we attempt to recreate these results for general sphere bundles, even though they lack the unfolding structure.

**A new proof of the existence of definable Whitney stratifications.**

David Trotman (Aix-Marseille University)

We give a geometric proof of existence of Whitney stratifications of definable sets in o-minimal structures, inspired by work of Kaloshin.

Joint work with Nhan Nguyen and Saurabh Trivedi.

**The Reeb graph of a map germ from $\mathbb{R}^3$ to $\mathbb{R}^2$**

Erica Boizan Batista (Universidade Estadual Paulista - UNESP)

The topological type of a finitely determined map germ $f : (\mathbb{R}^3, 0) \to (\mathbb{R}^2, 0)$ is given by the so-called link of $f$. The link of $f$ is obtained by taking a small enough representative $f : U \subset \mathbb{R}^3 \to \mathbb{R}^2$ and the intersection of its image with a small enough sphere $S^1_\varepsilon$ centered at the origin in $\mathbb{R}^2$. As a consequence of Fukuda’s theorem, two finitely determined map germs $f, g : (\mathbb{R}^3, 0) \to (\mathbb{R}^2, 0)$ are topologically equivalent if their associated links are topologically equivalent.

Inspired by the works of Arnold, Maksymenko and Prishlyak we introduce an adapted version of the Reeb graph that turns out to be a complete topological invariant for the links.

We take special attention to the case where $f$ has corank 1. Moreover, we give a complete description of those map germs with Boardman symbol $\Sigma^{2,1}$ and we provide a complete topological classification of this type of map germs up to multiplicity 6.

Joint work with João Carlos Ferreira Costa and Juan Jose Nuño Ballesteros.

**The critical points of a polar weighted homogeneous polynomial**
The Milnor's fibration theorem is very important in singularity theory, this result gives us information about the topology of the fibers of analytic functions near their critical points. This theorem for real analytic maps \( f : \mathbb{R}^{m+k} \to \mathbb{R}^k \) asks as a hypothesis that \( f \) has an isolated critical point in a neighbourhood of \( 0 \in \mathbb{R}^{m+k} \). A problem about this theorem is that examples are difficult to find, so one may ask: Under which conditions a real analytic map has a Milnor's fibration?

The first non-trivial example was given by A\'Campo, later Ruas-Seade-Verjovsky give a family of examples and J.L. Cisneros-Molina introduces the family of the polar weighted homogeneous polynomials, which are a generalization of complex weighted homogeneous polynomials. This family of examples has isolated critical value and has a Milnor's fibration with projection map \( f \) |

A natural question is: Given \( f \) a polar weighted homogeneous polynomial, is it possible to find conditions in order that \( f \) has isolated critical point? An answer to this question will provide examples of real analytic functions with a Milnor's fibration with projection map \( f \) |

The aim of this talk is to give a panoramic survey on polar weighted homogeneous polynomials, construct some examples in order to show the differences with the complex case and give a partial answer to the aforementioned question.

**Symplectic singularities of quasi-homogeneous curves**

Fausto Assunção de Brito Lira (ICMC-USP)

In this presentation we talk about symplectic classification of parameterized quasi-homogeneous curve-germs using the method of algebraic restrictions introduced by W.Domitrz, S.Janeczko and M. Zhitomirskii . We also obtain the symplectic classification of all curve-germs with semigroup \((4, 5, 6, 7)\).

Joint work with Roberta Wik-Atique and W.Domitrz.

**Minkowski caustics and symmetry sets.**

Graham Reeve (ICMC-USP)

I shall talk about the latest developments relating to Minkowski caustics and Minkowski symmetry sets. These are Minkowski analogues of the Euclidean versions.

**Multiple points of singular maps**

Guillermo Peñafort Sanchis (Universitat de Valencia)

We will study three different algebraic structures, due to Gaffney and Mond, which define multiple points of maps. We will show that they coincide in different situations, show a counterexample of
their equality and extend one of them to define multiple points of a map of any corank, possibly with singular source.

**Spacelike slant geometry on spacelike submanifolds of codimension two**

Handan Yıldırım (Istanbul University)

In this talk, we construct spacelike slant geometry on spacelike submanifolds of codimension two in Lorentz-Minkowski space from the viewpoint of Lagrangian singularity theory, [5]. Thus, we generalize some of the results which were obtained in [2]. Moreover, we interpret the results which were given in [1], [3] and [4] as special cases of our results. Furthermore, we mention about spacelike curves in Lorentz-Minkowski 3-space as another special case of our results.

Joint work with Shyuichi Izumiya.

**References**


**Cyclic suspensions of real singularities**

Haydee Aguilar Cabrera (ICMC, USP)

In this talk we will present some results about the topology of the family of real analytic germs $F: (\mathbb{C}^{n+1}, 0) \to (\mathbb{C}, 0)$ with isolated critical point at the origin, given by $F(x, y, z) = f(x, y) + z^r$, where $f$ is a real analytic function, $r \in \mathbb{Z}^+$ e $r \geq 2$.

**Classification of Morse Bott functions on surfaces.**

Ingrid Sofia Meza Sarmiento (ICMC-USP)

Let $S$ be a closed smooth surface and $f, g : S \to \mathbb{R}$ smooth functions with a finite number of critical points, then $f, g$ are called topologically equivalent if there are homeomorphisms $h : S \to S$ and $l : \mathbb{R} \to \mathbb{R}$ such that $f \circ h = h \circ g$. 
In Singularity Theory it is usual to consider differentiable equivalence and the local classification is the most studied. Here we analyze global classification.

Some previous work are [1], where Arnold says: The topological structures of the generic smooth functions on a smooth manifold belong to the small quantity of the most fundamental objects of study both in pure and applied mathematics.

The complete classification of Morse functions on closed surfaces was obtained by Sharko [4] and Kulinich [2]. The case of circle valued functions is considered in [3].

The purpose of this talk is to present the topological classification of Morse Bott functions defined on closed surfaces. Recall that a function $f : S \to \mathbb{R}$ is called Morse Bott function if the critical points sets are organized in nondegenerate smooth critical submanifolds. A critical submanifold $N$ of $f$ is called nondegenerate if the Hessian of $f$ is nondegenerate on each normal direction to $N$. For us, the Morse Bott functions can be real or circle valued functions. The basic tools in our study are: the Reeb graph and the classification of singular level sets up to homeomorphism.

Joint work with Jose Martínez Alfaro and Regilene Delazari Dos Santos Oliveira.

References


Equisingularity in complex surfaces

Jawad Snoussi (UNAM)

We explore some equisingularity criteria in one parameter families of generically reduced curves. We prove the equivalence between Whitney regularity and Zariski discriminant criterion. We prove that topological triviality implies smoothness of the normalized surface. Examples are given to show that Whitney regularity and equisaturation are not stable under the blow-up of the singular locus nor under the Nash modification.

Joint work with Javier Fernandez de Bobadilla and Mark Spivakovsky

TBA

Jean-Jacques Risler (PARIS 6)
On the topology of real analytic maps
Jose Luis Cisneros Molina (Universidad Nacional Autonoma de Mexico)
We study the topology of the fibres of real analytic maps $\mathbb{R}^n \to \mathbb{R}^p$, $n > p$, in a neighbourhood of a critical point. We first prove that every real analytic map germ $f: \mathbb{R}^n \to \mathbb{R}^p$, $p \geq 1$, with arbitrary critical set, has a Milnor-Lê type fibration away from the discriminant. Now assume also that $f$ has the Thom $a_f$-property, and its zero-locus has positive dimension. Also consider another real analytic map germ $g: \mathbb{R}^n \to \mathbb{R}^k$ with an isolated critical point at the origin. We have Milnor-Lê type fibrations for $f$ and for $(f, g): \mathbb{R}^n \to \mathbb{R}^{p+k}$, and we prove for these the analogous of the classical Lê-Greuel formula, expressing the difference of the Euler characteristics of the fibres $F_f$ and $F_{f,g}$ in terms of an invariant associated to these maps.
Joint work with Nivaldo G. Grulha Jr. and Jose Seade.

Topological triviality of families of map germs from $\mathbb{R}^3$ to $\mathbb{R}^3$
Juan Antonio Moya Pérez (Universitat de València)
We show that a 1-parameter unfolding $F: (\mathbb{R}^3 \times \mathbb{R}, 0) \to (\mathbb{R}^3 \times \mathbb{R}, 0)$ of a finitely determined map germ $f$, with $S(f)$ being regular, is topologically trivial if it is excellent in the sense of Gaffney and the family of the double point curves and cuspidal edges $D(f_i) \cup C(f_i)$ is topologically trivial.

Realizing local moves of the Stein factorizations of smooth maps from 3-manifolds to 2-manifolds
Kazuto Takao (Kyushu University)
The Stein factorization is useful to describe the topological behavior of a smooth map. In particular, that of a smooth map from a 3-manifold to a 2-manifold is generically a 2-dimensional cell-complex. In this talk, I give some local moves of the Stein factorization which can always be realized by a homotopy of the map.

Multisections of Lefschetz fibrations and mapping class groups of surfaces
Kenta Hayano (Hokkaido University)
A Lefschetz fibration is a smooth map from a four-manifold to a surface which is equivalent to the germ $(z_1, z_2) \mapsto z_1^2 + z_2^2$ at any critical point. Donaldson proved that every symplectic four-manifold can admit a Lefschetz fibration after blow-ups, and Usher related multisections of Lefschetz fibrations to the Gromov invariant for symplectic manifolds. In this talk we will explain how to see topology and configuration of multisections of Lefschetz fibrations in terms of mapping class groups of surfaces. As an application we will give new examples of Lefschetz fibrations with multisections.
Joint work with Refik İnanç Baykur (University of Massachusetts).
Criteria and geometries of Morin singularities

Kentaro Saji (Kobe university)

Morin singularities are the map-germs \((\mathbb{R}^m, 0) \rightarrow (\mathbb{R}^n, 0)\) which are \(A\)-equivalent to

\[
(x_1, \ldots, x_{m-1}, z) \mapsto (x_1, \ldots, x_{m-1}, g_1(x_1, \ldots, x_{m-1}, z), \ldots, g_{n-m+1}(x_1, \ldots, x_{m-1}, z)),
\]

where

\[
g_i(x_1, \ldots, x_{m-1}, z) = \sum_{j=1}^{k} x_{(i-1)k+j} z^j \quad (i = 1, \ldots, n-m),
\]

and

\[
g_{n-m+1}(x_1, \ldots, x_{m-1}, z) = \sum_{j=1}^{k-1} x_{(n-m)k+j} z^j + z^{k+1},
\]

when \(m < n\), or

\[
(x_1, \ldots, x_{n-1}, y_1, \ldots, y_{m-n}, z) \mapsto \left( x_1, \ldots, x_{n-1}, q(y_1, \ldots, y_{m-n}) + z^{k+1} + \sum_{i=1}^{k-1} x_i z^i \right).
\]

where \(q(y_1, \ldots, y_{m-n}) = \sum_{i=1}^{m-n} p_m y_i^2\), when \(m \geq n\). They are fundamental singularities of \(C^\infty\)-maps. In this talk, first I give a useful criteria for Morin singularities, and using them, I study geometric and topological properties of Morin map-germs.

Pairs of Hypersurfaces in Volume Preserving Geometry

Konstantinos Koufliouros (Imperial College London)

In this talk I will consider the problem of analytic classification of a pair of hypersurfaces with respect to a volume form. I will study only the case where one of the hypersurfaces is smooth and the other has at most isolated singularities, so that the pair of hypersurfaces defines an isolated boundary singularity in the sense of V. I. Arnol’d. In particular, extending some classical results of the Gauss-Manin theory of isolated singularities in the presence of a hypersurface (boundary), I will prove a generalisation of a theorem of A. N. Varchenko, i.e. that the number of moduli appearing in the classification of the triples (volume form, pair of hypersurfaces) is exactly equal to the degree of non-quasihomogeneity of the corresponding boundary singularity.

When is a complex surface in \(\mathbb{C}^3\) is a topological manifold?

Lê Dũng Tráng (Universidade Federal do Ceará)

In this lecture we shall relate the notion of equisingularity and the fact that it is a topological manifold for a complex surface in \(\mathbb{C}^3\).

Quartics and Quintics a review

Leon Kushner (Universidad Nacional Autonoma de Mexico)
In this research we are making a review of homogeneous polynomials of degree 4 and 5 in two variables. After working with the new program in the computer and with a better understanding of the problem we get better results. The task is the classification of the homogeneous polynomials of degree four and five and their stabilizers. As before there is not a complete relation between the highest dimension of the orbits which is at most 4 and finite determination.

**First integrals for generic singularities of complex vector fields in dimension 3**

Leonardo Câmara (Universidade Federal do Espírito Santo)

This talk is about the existence of first integrals for singularities of holomorphic vector fields in complex dimension three. Under generic conditions, we prove a topological criteria for the existence of a holomorphic first integral. Our result may be seen as a kind of Reeb stability result for the framework of vector fields singularities in dimension three. As a consequence, we prove that, for the class of singularities we consider, the existence of a holomorphic first integral is invariant under topological equivalence.

Joint work with Bruno Scárdua.

**Affine metrics of locally strictly convex surfaces in affine 4-space**

Luis Florial Espinoza Sanchez (ICMC-USP)

We introduce a new family of affine metrics on a locally strictly convex surface $M$ in affine 4-space. Then, we define the symmetric and antisymmetric equiaffine planes associated with each metric. We show that if $M$ is immersed in a locally strictly convex hyperquadric, then the symmetric and the antisymmetric planes coincide and contain the affine normal to the hyperquadric.

**Iteration of Involutes of Constant Width Curves in the Minkowski Plane**

Marcos Craizer (Catholic University- Rio de Janeiro)

In this talk we consider properties of the area evolute (AE) and the center symmetry set (CSS) of a convex planar curve $\gamma$. The main tool is to define a Minkowski plane where $\gamma$ becomes a constant width curve. In this Minkowski plane, the CSS is the evolute of $\gamma$ and the AE is an involute of the CSS. We prove that the AE is contained in the region bounded by the CSS and has smaller signed area.

The iteration of involutes generate a pair of sequences of constant width curves with respect to the Minkowski metric and its dual, respectively. We show that these sequences are converging to symmetric curves with the same center, which can be regarded as a central point of the curve $\gamma$. 


Critical points of a smooth map and its applications

Maria García Monera (Universitat Politècnica de València)

We introduce the concept of \( r \)-critical point of a smooth map between manifolds, and apply it to the study of \( 1, 2 \)-critical points of a smooth map.

Jumps of Milnor numbers

Maria Michalska (University of Lodz)

The aim of this talk is to show the possible Milnor numbers of deformations of semi-quasi-homogeneous isolated plane curve singularities. Main result states that if a singularity \( f \) is irreducible, by deforming \( f \) one can attain all Milnor numbers ranging from \( \mu(f) \) to \( \mu(f) - r(p - r) \), where \( r \) and \( p \) are easily computed from the Newton diagram of the singularity \( f \).

Joint work with Justyna Walewska.

Parabolic curves on zero sets of functions with a singularity

Masaru Hasegawa (ICMC - USP)

Let \( f : \mathbb{R}^3, 0 \to \mathbb{R}, 0 \) be a germ of smooth function with a simple singularity. We investigate parabolic curves on the zero set of \( f \).

Joint work with Farid Tari.

Evolutes of curves in the Lorentz-Minkowski plane

Masatomo Takahashi (Muroran Institute of Technology)

We can use a moving frame in order to define the arc-length parameter and the Frenet formulae for non-lightlike regular curves in the Lorentz-Minkowski plane. This leads naturally to a well defined evolute associated to non-lightlike regular curves without inflection points in the Lorentz-Minkowski plane. However, at a lightlike point the curve shifts between a spacelike and a timelike region and the evolute cannot be defined by using this moving frame. We introduce an alternative frame, the lightcone frame, that will allow us to associate an evolute to regular curves without inflection points in the Lorentz-Minkowski plane. Moreover, under appropriate conditions, we shall also be able to obtain globally defined evolutes of regular curves with inflection points. This is a joint work with S. Izumiya and M.C. Romero Fuster.

On a mixed deformation of isolated singularities and the number of cusps

Masayuki Kawashima (Tokyo university of science)
It is known that the number of cusps of mixed deformation of Morse singularities. In this talk, we generalize this result for other isolated singularities.

**Hilbert modular cusps and Lutz-Mori twists**

Naohiko Kasuya (Tokyo University)

We detect the canonical contact structure on the link of a complex surface singularity \( x^p + y^q + z^r + xyz = 0 \) \((p^{-1} + q^{-1} + r^{-1} < 1)\) by two different methods. One is a toric method, and the other is a method using Hilber modular cusps. We also explain the relation between Hilbert modular cusps and the Lutz-Mori twist, which is a generalization of the Lutz-twist to higher dimensional contact geometry defined by Atsuhide Mori.

**La méthode des façons: a new way to stratify the asymptotic set associated to a polynomial map**

Nguyen Thi Bich Thuy (ICMC-USP)

In the 90s, Zbigniew Jelonek showed that the asymptotic set of a polynomial mapping \( F : \mathbb{C}^n \to \mathbb{C}^n \) is a \((n-1)\) - (complex) dimensional singular variety. We give a method, called méthode des façons, for stratifying this set. We obtain a Thom-Mather stratification. As application, we can compute the intersection homology of a set that we call the Valette set \( V_F \). In 2010, Anna and Guillaume Valette constructed a real pseudomanifold \( V_F \subset \mathbb{R}^{2n+p} \), where \( p > 0 \), associated to a polynomial mapping \( F : \mathbb{C}^n \to \mathbb{C}^n \). In the case \( n = 2 \), they proved that if \( F \) is a polynomial mapping with nowhere vanishing Jacobian, then \( F \) is not proper if and only if the homology (or intersection homology) of \( V_F \) is not trivial in dimension 2. We give also a generalization of this result, in the case of a polynomial mapping \( F : \mathbb{C}^n \to \mathbb{C}^n \) with nowhere vanishing Jacobian. By using the façons, we give an algorithm for expliciting the asymptotic sets of a dominant quadratic polynomial mapping in three variables. As a result, we have a complete list of the asymptotic sets in this case.

**Lê-Greuel type formula for the Euler obstruction of a function**

Nivaldo de Goes Grulha Junior (ICMC-USP)

The Euler obstruction of a function \( f \) can be viewed as a generalization of the Milnor number for functions defined on singular spaces. In this work, using the Euler obstruction of a function, we give a version of the Lê-Greuel formula for germs \( f : (X, 0) \to (\mathbb{C}, 0) \) and \( g : (X, 0) \to (\mathbb{C}, 0) \).

Joint work with N. Dutertre.

**The center problem for a 1:-4 resonant quadratic system**
Regilene Oliveira (ICMC-USP)

The main object of this paper is to find necessary and sufficient conditions for a $1: -4$ resonant system of the form

$$\dot{x} = x - a_{10}x^2 - a_{01}xy - a_{12}y^2$$

$$\dot{y} = -4y + b_{2,-1}x^2 + b_{10}xy + b_{01}y^2$$

to have a center at the origin. The necessary conditions are obtained using modular arithmetics. The sufficiency of each obtained condition is proven using a local analytic first integral - to find it or prove its existence distinct criteria are used.

Joint work with Brigita Ferčec (CAMPT-Slovenian), Jaume Gine (Universitat de Lleida-Spain and Matej Mencinger (University of Maribor, Slovenian).

**Theory of wave front propagations and its applications**

Shuichi Izumiya (Hokkaido University)

In the workshop I will summarize the overview of the school talk of the next week. I will mainly explain some applications of the theory of wavefront propagations which I will not be able to talk at the school talk.

**Splitting of singular fibers and topological monodromies**

Takayuki OKUDA (Kyushu University)

It is known that there is a good relation between the topological types of degenerations of Riemann surfaces and the surface mapping classes, via topological monodromy. In this talk, we are interested in splitting families for degenerations, that is, deformation families which split the original singular fiber into several simpler singular fibers. We want to introduce the topological monodromies of splitting families and show that they play an important role on studying the topology of splitting families.

**Topological Poincar’e series of normal surface singularities and Ehrhart theory of polytopes**

Tamás László (MTA Renyi Institute of Mathematics)

In this talk let $M$ be the link of a normal surface singularity and we assume it is a rational homology sphere. In other words, $M$ is a plumbed 3-manifold associated with a connected negative definite plumbing tree with everywhere zero genus decorations.

We show that its Seiberg-Witten invariants equal certain coefficients of an equivariant multivariable Ehrhart polynomial. For this, we construct the corresponding polytope from the plumbing graph together with an action of $H_1(M; \mathbb{Z})$ and we develop Ehrhart theory for them.
At an intermediate level we define the multivariable topological Poincaré series associated with the graph. One can introduce the generalized periodic constant and for its calculation we can reduce the variables of the series. In this way, one identifies the Seiberg-Witten invariant of a plumed 3-manifold, the periodic constant of its Poincaré series, and a coefficient of the associated Ehrhart polynomial. We discuss the process for graphs with \( n \) nodes. It has surprising connections with the theory of affine monoids of rank \( n \).

The equisingularity of codimension 2 Cohen-Macaulay isolated singularities

Terence Gaffney (Northeastern University)

In previous works I have constructed a framework for approaching problems in equisingularity through invariants computed from pairs of modules associated to the singularity. In this talk I will discuss recent improvements to this framework, from both an intersection theoretic and length theoretic approach, that apply to families of codimension 2 Cohen-Macaulay isolated singularities.

Joint work with my student Antoni Rangachev.

Is the module of lowerable vector fields finitely generated?

Yusuke Mizota (Kyushu University)

The notion of lowerable vector field was introduced by Arnold for studying bifurcations of wave front singularities. Is the module of lowerable vector fields finitely generated in general? In this talk, we give a partial affirmative answer to this problem.

Joint work with T. Nishimura.

Reference


Recognition of map-germs from 3-space to 3-space and applications

Yutaro Kabata (Hokkaido University)

We show some criteria for map-germs from 3-space to 3-space of corank one. Then we show applications of our criteria to geometric settings such as classification of singularities in line congruences.
1.4. Posters.

Envelope of Mid-Planes of Surfaces

Ady Cambraia Junior (Pont. Univers. Católica - Rio de Janeiro)

The envelope of the mid-lines of convex plane curves is a subject that has been extensively explored. It consists of the union of the following affine invariant sets: the Affine Envelope Symmetry Set-AESS (non-parallel tangent lines), the Mid-Parallel Tangents Locus- MPTL (parallel tangent lines) and Affine Evolute (coincident tangent lines) [1],[2]. In this poster, I propose to present a generalization of the envelope of the mid-lines of curves for surfaces.

References


Topological classification and stems of co-rank two map germs from the plane to the plane

Aldício José Miranda (Universidade Federal de Uberlândia)

We investigate the topological classes in $\mathcal{K}$-orbits of co-rank two finitely determined complex map germs from the plane to the plane. To describe these orbits we use the topological invariants, the number of vanishing cusps and double folds, which appear in a generic deformation of such germs. First we show how to obtain the number of vanishing cusps in terms of Newton non degeneracy conditions of the critical curve of the germ. To study the double folds we use the relations among these invariants and the Fitting ideals. We show here a method to obtain all topological classes of co-rank two complex germs from the plane to the plane, to illustrate how this method works we investigate the topological orbits in the $\mathcal{K}$-class $(xy, x^3 + y^4)$. Moreover we give the complete answer for a question proposed by Gaffney and Mond about the number of topological orbits in a given $\mathcal{K}$ orbit. The key tool to answer this question is the existence of some special germs called it stems by D. Mond, these germs are in a given $\mathcal{K}$-orbit and are not $A$-finitely determined.

Joint work with M. J. Saia and L. M. F. Soares.

Generic local structure of curves and surfaces by unfoldings of real functions

Alex Paulo Francisco and Luciana F. Martins (IBILCE - UNESP)

Let $F : \mathbb{R} \times \mathbb{R}^r \to \mathbb{R}$ be a smooth function. We can naturally regard $F$ as an $r$-parameter family of functions, which is called an unfolding of a certain function in this family. The existence of unfoldings with the property of be “versal” is one of the central results of the Singularity Theory. Roughly speaking, a versal unfolding of a real function $g$ contains every functions close to $g$. 
Recognize versal unfoldings is important to study properties of subsets of the parameter's space which are preserved by diffeomorphisms. The main goal of this work is present some results about unfoldings of real functions and some applications to the study of the generic local structure of curves and surfaces.

We know that curves and surfaces are, in neighborhoods of regular points, diffeomorphic to lines and planes, respectively. Therefore, the question that remains to be analyzed is regarding the local structure on singular points, that is, on points where there are not a tangent line or a tangent plane well-defined. In this case, we will use the concepts and results about unfoldings of real functions to conclude about the generic local structure of some curves and surfaces, like the evolute, the parallel curve and surface, the orthotomic, among others.

In this presentation we will go through some of the results that are in the master dissertation of the first author under supervision of Luciana F. Martins. The book Curves and Singularities: a geometrical introduction to singularity theory 2nd ed. Cambridge University Press (1992) of J.W. Bruce and P.J. Giblin is our main reference.

TBA

Camila Mariana Ruiz (ICMC-USP)

Global study of maps between closed surfaces

Catarina mendes de Jesus (Federal Universtity Viçosa)

Second H. Withney, the stable maps between surfaces can only have fold singularities along curves with isolated cusp singularities. In order to codify the topology of the regular set of stable maps between closed surfaces, in previous works, we introduced global invariant, known as the graph of a stable map and we studied the properties of these graphs and their behavior through convenient surgeries and isotopies of maps from a closed orientable surface to the sphere and we determined the properties of the graphs that can be associated to stable maps for each fixed degree d. Here we extend the result to stable maps between any two closed orientable surfaces.

A note on the Mond conjecture and crosscap concatenations

Catiana Casonatto (Universidade Federal de Uberlândia - UFU)

We prove the Mond conjecture relating the codimension of a map germ from \(\mathbb{C}^n\) to \(\mathbb{C}^{n+1}\) with its image Milnor number for multigerms resulting from the operation of simultaneous augmentation and monic concatenation. We then define a new operation, the crosscap concatenation, in order to obtain new examples of multigerms where the Mond conjecture can be tested.

Joint work with Raúl Oset Sinha.
TBA

Douglas Hilário da Cruz (Universidade Federal de Goiás)

TBA

Remarks about Discriminants of map germs from $\mathbb{C}^{n+1}$ to $\mathbb{C}^n$, $n = 3, 4$.

Eliris Cristina Rizziolli (IGCE-UNESP-Rio Claro/SP)

In this work we show some calculations involving Lê numbers and Euler characteristic of the Milnor fibre on discriminants of map germs from $\mathbb{C}^{n+1}$ to $\mathbb{C}^n$, $n = 3, 4$.

Joint work with M. J. Saia.

Normal Forms of pairs of involutions with hyperbolic normal composition

Elizabeth Ruth Salazar Flores (UFRJ)

This work proposes to address the study of involutions for the research into Dynamical Systems. More precisely, we treat the study of the question of conditions for simultaneous linearization of the class of pairs of involutions with hyperbolic normal composition and the deduction of the normal forms of these pairs and normal forms of the case hyperbolic.

Realization of graphs by fold Gauss maps

Esther Sanabria Codesal (Universitat Politècnica de València)

We can associate graphs that codify the topological type of singular and regular sets of Gauss stable maps. In this work we determine family of graphs that can be associated to Gauss fold maps from closed orientable surfaces. We determine necessary and sufficient condition that tells us when some graph can be the graph of a Gauss fold map for the particular case of the 2-sphere.

Joint work with Catarina Mendes de Jesus.

Second order invariants of a surface immersed in $\mathbb{R}^n$, $n > 4$.

Felipe de Jesús Méndez Varela

It is known that for a surface immersed in $\mathbb{R}^n$, the curvature ellipse, a configuration formed by a point and an ellipse gives a classification for the second fundamental form. This configuration is in the normal space and determines the second order invariants. This work describes an invariant system that is (almost) equivalent to those already known and the existing relation between them and the curvature ellipse. These invariants lets recover the second fundamental form they are associated to, up to an action of a certain group $G$. Finally it is given a sufficient condition in terms of the invariants for a surface immersed in $\mathbb{R}^n$ to be locally convex.
Foliations and global injectivity of local diffeomorphisms in $\mathbb{R}^3$

Jean Venato Santos (Universidade Federal de Uberlândia)

We consider $C^\infty$ local diffeomorphism $F = (F_1, F_2, F_3) : \mathbb{R}^3 \to \mathbb{R}^3$. We prove that each of the following conditions are sufficient to the global injectivity of $F$:

- The foliations $F_{F_i}$ made up by the connected components of the level surfaces $F_i =$ constant, consist of leaves without half-Reeb components induced by $F_j$, $j \in 1, 2, 3 \setminus i$, for $i \in 1, 2, 3$. For each $i \geq j \in 1, 2, 3$, $F_i|_L : L \to R$ satisfy the Palais-Smale condition, for all $L \cap F_j$.

We also prove that B implies A and give examples to show that the converse is not true. Further, we give examples showing neither of the conditions are necessary to the global injectivity of $F$.

Joint work with Francisco Braun (UFSCar)

Familias normales y grupos discontinuos

Jimmy Rainer Tamara Albino (Pontificia Universidad Católica del Perú)

Mattei-Moussu with and without explosions

Jonny Ardila Ardila (Federal University of Rio de Janeiro)

In this poster I will show the heart of two proofs of Mattei-Moussu theorem, the first is the one in the original article, and the second is a more geometric one due to Moussu. I will explain the difference between both, their limitations and how we are rewriting (reformulating) the second proof to find new extensions or applications of the result.

Multi-local singularities of orthogonal projections of surfaces in $\mathbb{R}^4$ to 3-spaces.

Jorge Luiz Deolindo Silva (Universidade de São Paulo)

We study the geometry of surfaces in $\mathbb{R}^4$ associated to contact with lines. This contact is captured by the local and multi-local singularities of the orthogonal projections to 3-space. The local singularities have been extensively studied. Here we deal with the multi-local singularities. In
particular, we show the existence of (multi-local) robust feature of surface. These are smooth curves representing the various types of multilocal singularities.

Joint work with Farid Tari.

Detecting bifurcation values at infinity of real polynomials

Luis Renato Gonçalves Dias (Universidade Federal de Uberlândia)

We present a new approach for estimating the set of bifurcation values at infinity. This yields a significant shrinking of the number of coefficients in the recent algorithm introduced by Jelonek and Kurdyka for reaching critical values at infinity by rational arcs.

Joint work with M. Tibar.

On Segre numbers of homogeneous map germs

Michelle Ferreira Zanchetta Morgado (IBILCE - UNESP)

Segre numbers and Segre cycles of ideals were independently introduced by Gaffney and Gassler and by Tworzewski. They are generalization of the Lê numbers and Lê cycles, introduced by Massey. In this work is given Lê-Iomdine type formulas for these cycles and numbers of arbitrary ideals. As a consequence is given a Plucker type formula for the Segre numbers of ideals generated by weighted homogeneous functions, in terms of their weights and degree. As an application of these results, is computed, in a purely combinatorial manner, the Segre numbers of the ideal which defines the critical loci of a map germ defined by a sequence of central hyperplane arrangements in \( \mathbb{C}^{n+1} \).

Joint work with R. Callejas-Bedregal (UFPB) and M. J. Saia (ICMC-USP)

Contact of space curves with cylinders

Mostafa Salarinoghahi (ICMC-USP)

Let \( \gamma : I \to \mathbb{R}^3 \) be a space curve and \( \alpha \) be its orthogonal projection along a unit direction \( v \). We study the contact of \( \alpha \) with circles in the plane orthogonal to \( v \). This gives information about the contact of \( \gamma \) with cylinders.

Invariants of Determinantal Varieties

Nancy Carolina Chachapoyas Siesquen (Universidade de São Paulo)

In this work, we study the essentially isolated determinantal singularities (EIDS), which have been defined in the article by S. M. Gusein-Zade and V. Ebeling, this type of singularities is as a natural generalization of isolates ones. The determinantal varieties are defined as the pre-image of a function holomorph, \( F : \mathbb{C}^N \to M_{m,n} \), on the generic determinantal variety, \( M_{m,n} \), where
t \leq \min\{n, m\} with the condition \( X = M^t_{m,n} \). It is known that this type of determinantal variety has isolated singularity if \( N \leq (n-t+2)(m-t+2) \) and admit smoothing if \( N < (n-t+2)(m-t+2) \) by Jonathan Wahl. Several recent works are studied the determinantal variety with isolated singularity and they have defined the Milnor number of a determinantal surface and the evanescent Euler characteristic for determinantal variety. We study the set of limits of tangent hyperplane to determinantal variety \( X^2 \subset \mathbb{C}^4 \) and \( X^3 \subset \mathbb{C}^5 \) for give a characterization of these hyperplane, by the fact that the Milnor number of her section with the surface in the first case (or the 3-dimensional determinantal variety in the second case) is not minimum. The first case is studied by Jawad Snoussi. We prove too that if \( X \) is a EIDS and \( H \) and \( H' \) are general strongly hyperplane, the Milnor number of the surfaces \( X \cap P \) et \( X \cap P' \) are equals if \( P \subset H \) and \( P' \subset H' \) are linear planes contained in \( H \) et \( H' \). This result has been showed in the case that the section by Lê Dũng Tráng.

Joint work with Maria Aparecida Soares Ruas and Jean-Paul Brasselet.

A connection between the Newtonian two–body problem and slow–fast systems

Pedro Tonio Cardin (UNESP)

In this work we consider the Newtonian two–body problem

\[
\begin{align*}
\ddot{y}_1(t) &= -Gm_2 \frac{y_1 - y_2}{|y_1 - y_2|^3}, \\
\ddot{y}_2(t) &= -Gm_1 \frac{y_2 - y_1}{|y_1 - y_2|^3},
\end{align*}
\]

where \( y_i(t) \in \mathbb{R}^3 \), \( i = 1, 2 \), are the positions of two particles of masses \( m_i \) and \( G \) is the gravitational constant. We show that the dynamics of \((??)\) is equivalent to the dynamics of a slow–fast system.

Knots and isolated singularities of Surfaces in 4-space

Rodrigo Mendes Pereira (Universidade Federal do Ceará)

Any finitely determined map germ \( \mathbb{R}^n \to \mathbb{R}^p \) has a cone structure over its link by Fukuda’s theorem. The link is obtained by taking the intersection of its image with a small enough \( (p-1) \)-dimensional sphere. Associated with this link (which turns out to be a stable map), we have a complete topological classification for the case \( n = 2 \) and \( p = 3 \) via the existence of Gauss words (by the results of Nuño-Marar). In this paper, we present the ideas in order to start the study of the case \( n = 2, p = 4 \), where the link is now a knot in \( \mathbb{S}^3 \). We prove, in particular, that a convenient projection of \( \mathbb{S}^3 \) in \( \mathbb{S}^2 \), can be obtained by taking a generic projection from \( \mathbb{R}^4 \) to \( \mathbb{R}^3 \). Thus, for any map germ \( \mathbb{R}^2 \to \mathbb{R}^4 \) of type \( \sum^{1,0} \), its link is the trivial knot. We expect a reasonable classification considering generic projections on the link \( \mathbb{S}^3 \) in \( \mathbb{S}^2 \) by looking at its associated Gauss word.

Joint work with J. J. Nuño Ballesteros

TBA
Characterization of trivial map germs for real Milnor fibers in dimensions $(6, 3)$.

Taciana Oliveira Souza (Universidade Federal de Uberlândia)

In the book “Singular points of complex hypersurfaces” [1], John W. Milnor studied singular points on hypersurfaces introducing a locally trivial fiber bundle, called the Milnor fibration, associated to each singular point. He showed the existence of such structures for germs of holomorphic functions and real analytic maps. We are interested specially in the case where Milnor considered a real polynomial mapping $f : (\mathbb{R}^n, 0) \rightarrow (\mathbb{R}^p, 0)$. Milnor proposed to call the singularity emphtrivial if the fiber of the Milnor fibration associated is diffeomorphic to the disk, and he asked [1]: “For which dimensions $n \geq p \geq 2$ do non-trivial examples exist?”

Church and Lamotke answered this question in [2]. In [3], we extend the characterization of trivial map germs for the real Milnor fibration, started by Church and Lamotke, in dimensions $(n, p) = (6, 3)$. For $(n, p) = (6, 3)$ if the singularity is non-trivial then the Milnor fiber associated is homotopy equivalent to a bouquet of 2-spheres, but a similar result is not true in general.

Joint work with: M. A. B. Hohlenwerger and R. Araújo dos Santos.

References


Generalized distance-squared mappings of the plane into the plane

Takashi Nishimura (Yokohama National University)

We define generalized distance-squared mappings, and we concentrate on the plane to plane case. We classify generalized distance-squared mappings of the plane into the plane in a recognizable way.
Joint work with S. Ichiki, R. Oset Sinha and M. A. S. Ruas.

The Euler Obstruction and Torus Action
Thais M. Dalbelo (ICMC-USP)

In this work we study surfaces with the property that their irreducible components are toric surfaces. In particular, we present a formula to compute the Euler obstruction of such surfaces. As an application of this formula we compute the Euler obstruction for some families of determinantal surfaces. We also make some remarks about Milnor number and toric surfaces in $\mathbb{C}^3$ and $\mathbb{C}^4$.

Joint work with N. G. Grulha Jr.
2. School on Singularity Theory

2.1. Mini-courses.

Analytic Classification of Plane Branches: An algebraic viewpoint
Marcelo Escudeiro (Universidade Estadual de Maringá)

The purpose of this mini-course is to present analytic (and topological) invariants for analytic irreducible plane curves and algebraic methods to compute them. In particular, we will describe normal forms for plane branches with respect to the analytical equivalence in a fixed topological class.

Theory of wave front propagations and Lorentz differential geometry
Shyuichi Izumiya (Hokkaido University)

The school talk will be separated into two parts.

I) The theory of wave front propagations.

We start to introduce the theory of wave front propagations which had been founded by Zarkalyukin [4,5]. Now it is known as the theory of big fronts (or, big Legendrian submanifolds). Later on, I introduced the notion of graph-like Legendrian unfoldings which belong to a special class of big Legendrian submanifolds [1]. Recently, we (with Masatomo Takahashi) have discovered some interesting relations of graph-like Legendrian unfoldings with caustics and Maxwel sets of corresponding Lagrangian submanifolds [2,3].

II) Applications to Lorentz differential geometry.

We apply the theory of graph-like Legendrian unfoldings to Lorentz differential geometry of submanifolds in Lorentz space forms (i.e., Minkowski space-time, de Sitter space or anti-de Sitter space). Since we do not have the notion of constant time in the relativity theory, we have to always consider one-parameter families of submanifolds depending on the time-parameter (i.e., world sheets). In this case, the submanifold with the constant parameter is not necessarily the constant time in the ambient space. Therefore the theory of wave front propagations is especially important for applications to Lorentz differential geometry.

References


Non-isolated Hypersurface Singularities, Stratifications, and Lê Cycles

David B. Massey (Northeastern University, Boston, USA)

In this series of lectures, I will discuss results for complex hypersurfaces with non-isolated singularities.

In Lecture 1, I will review basic definitions and results on complex hypersurfaces, and then present classical material on the Milnor fiber and fibration.

In Lecture 2, I will present basic results from Morse theory, and use them to prove some results about complex hypersurfaces, including a proof of Lê’s attaching result for Milnor fibers of non-isolated hypersurface singularities. This will include defining the relative polar curve.

Lecture 3 will begin with definitions and basic results on Lê cycles and Lê numbers of non-isolated hypersurface singularities; this will involve a brief discussion of intersection cycles for proper intersections inside a complex manifold. Then I will move onto more-complicated results.

Lecture 4 will continue with results on Lê cycles and numbers, including relatively recent results, like those of Bobadilla. If time permits, I will indicate the relationship between the Lê cycles and the complex of sheaves of vanishing cycles.

Milnor’s fibration theorems revisited

José Seade (UNAM, Cuernavaca, Mexico) and Jose Luis Cisneros-Molina (Universidad Nacional Autónoma de Mexico)

We’ll revisit the classical fibration theorems of Milnor for real and complex singularities, looking at these from a modern point of view. This is based on results by various people, obtained through decades of research on the topic. Then we’ll discuss some current trends of research on the subject.

2.2. Public lectures.

On a Riemann function

Marcio Soares (UFMG)

TBA

Exotic shapes of nano-spherical structures

Stanislaw Janeczko (Warsaw University of Technology)
Imaginary approach to rational explanation since Pitagoras, Plato and Archimedes is discusses. The simplest naturally ordered sphere packings are described and their special chain and cluster forms are classified. This is applied to form nanoparticles with well-defined chemical compositions.
3. Real and Complex Singularity Days of Young Researchers

3.1. 20 minutes talks.

\textbf{\textit{p}-adic Zeta Functions for Laurent Polynomials}

Edwin León Cardenal (UNAM)

This talk is about singularities over \( p \)-adic fields, but techniques involved are well known in the real case. We study local zeta functions attached to Laurent polynomials over \( p \)-adic fields, with the condition of being non-degenerate with respect to their Newton polytopes at infinity, at the end some applications are given. This is an introductory talk about the subject. Joint work with W. A. Zúñiga-Galindo.

\textbf{Second order invariants of a surface immersed in} \( \mathbb{R}^n \), \( n > 4 \).

Felipe de Jesús Méndez Varela (UNAM)

It is known that for a surface immersed in \( \mathbb{R}^n \), the curvature ellipse, a configuration formed by a point and an ellipse gives a classification for the second fundamental form. This configuration is in the normal space and determines the second order invariants. This work describes an invariant system that is (almost) equivalent to those already known and the existing relation between them and the curvature ellipse. These invariants lets recover the second fundamental form they are associated to, up to an action of a certain group \( G \). Finally it is given a sufficient condition in terms of the invariants for a surface immersed in \( \mathbb{R}^n \) to be locally convex.

\textbf{TBA}

Jane Lage Bretas (UFMG)

\textbf{Growth at infinity and index of polynomial maps}

Jorge Alberto Coripaca Huarcaya (ICMC-USP)

Let \( F : \mathbb{K}^n \to \mathbb{K}^p \) be a polynomial map such that \( F^{-1}(0) \) is compact, where \( \mathbb{K} = \mathbb{R} \) or \( \mathbb{C} \). In this work we give a lower bound for the Lojasiewicz exponent at infinity of \( F \). This bound is uniform in certain deformations of \( F \) and leads to a result about the invariance of the global index of \( F \).

\textbf{Multi-local aspects of surfaces in} \( \mathbb{R}^4 \).

Jorge Luiz Deolindo Silva (ICMC-USP)
We study the geometry of surfaces in $\mathbb{R}^4$ associated to contact with lines and hyperplane. This contact is captured by the local and multi-local singularities of the orthogonal projections to 3-space and the height functions. Here we deal with the multi-local singularities. In particular, we show the existence of (multi-local) robust feature of surface. These are smooth curves representing the various types of multilocal singularities.

Toric surfaces, vanishing Euler characteristic and Euler obstruction of a function

Miriam da Silva Pereira (Universidade Federal da Paraíba)

We define the vanishing Euler characteristic of $X_\sigma$ and give a formula to compute it and we relate it with the second polar multiplicity of $X_\sigma$. We also present a formula for the Euler obstruction of a function $f : X_\sigma \to \mathbb{C}$ and for the difference between the Euler obstruction of the space $X_\sigma$ and the Euler obstruction of a function $f$. As an application of this result we compute the Euler obstruction of a type of polynomial on a family of determinantal surfaces. Joint work with Thais Maria Dalbelo and Nivaldo Gruilha.

Metric properties of arcs at an isolated singular point

Rodrigo Mendes Pereira (Universidade Federal do Ceará)

Consider a subanalytic set of dimension 3 with an isolated isolated singularity. Consider and arc $\gamma$ going through the singular point and the family loops around a this arc collapsing to the singular point. This family is considered from the metric point of view. The contraction of these loops depends on the prescribed rates of vanishing. In this work we formalize such constructions. We also display strategies and speculation in order to obtain some finiteness results.

Joint work with Lev Birbrair.

Contact of a surface with cylinders and cubic binary differential equations

Toshizumi Fukui (Saitama University)

We investigate contact of a regular surface with cylinders in Euclidean 3-space. A classification result is given in generic context, We also remark a relation with a cubic binary differential equation.

Joint work with M. Hasegawa and K. Nakagawa.

Generalized open book structure for semialgebraic sets

Ying Chen (ICMC-USP)
Given a $C^2$ semi-algebraic mapping $F = (f_1, \ldots, f_p) : \mathbb{R}^N \to \mathbb{R}^p$ defined on a compact semi-algebraic manifold, we introduce sufficient conditions for the existence of a fibration structure induced by $\frac{E}{\|F\|}$. Moreover, the conditions we found can be applied to the local case too. Under these conditions, considering the composition mapping of $F$ with the canonical projection $\pi : \mathbb{R}^p \to \mathbb{R}^{p-1}$, we study the topology of fibre of $\frac{E}{\|F\|}$.

Joint work with Nicolas Dutertre, Raimundo Nonato Araújo dos Santos and Antonio Andrade do Espírito Santo.

Recognition of map-germs from plane to plane and applications
Yutaro Kabata (Hokkaido University)

We show a complete set of criteria for map-germs from plane to plane of corank one with $\mathcal{A}$-codimension $\leq 6$. Then we also show an application of our criteria to generic differential geometry.

3.2. Posters.

Topology of a holomorphic vector field in a neighborhood of an isolated singularity
Jonny Ardila Ardila (Federal University of Rio de Janeiro)

Taking advantage of the mini course *Non-isolated Hypersurface Singularities, Stratifications, and Lê Cycles*, I am gonna use this poster for introduce the article *On the topology of a holomorphic vector field in a neighborhood of an isolated singularity.* (Gomes-Mont, X. and Seade, J. and Verkhovskii, A.) which is very related to Milnor’s book, and I will try to show how this article and its continuation (Morse theory and the topology of holomorphic foliations near an isolated singularity - Limón, Beatriz and Seade, José) is been used in my current work, related with the existence of first integrals.

Lipschitz regular complex algebraic sets are smooth
José Edson Sampaio (IFCE)

The classical Theorem of Mumford states that a topologically regular complex algebraic surface in $\mathbb{C}^3$ with an isolated singular point is smooth. We proof that any Lipschitz regular complex algebraic set is smooth. No restriction on the dimension is needed. No restriction of singularity to be isolated is needed.

Joint work with Alexandre Fernandes, Lev Birbrair and Le Dung Trang.

Contact of space curves with cylinders
Mostafa Salarinoghabi (ICMC-USP)
See Workshop abstracts above.

**Invariants of Determinantal Varieties**

Nancy Chachapoyas Sesquien (ICMC-USP)

See Workshop abstracts above.
4. List of Participants

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