Theory of wave front propagations
and
Lorentz differential geometry

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Abstract

The talk will be separated into two parts.

I) The theory of wave front paropagations.

We start to introduce the theory of wave front propagations which had been founded by Zakalyukin [3]. Now it is known as the theory of big fronts (or, big Legendrian submanifolds). Later on, I introduced the notion of graph-like Legendrian unfoldings which belong to a special class of big Legendrian submanifolds[1]. Recently, we (with Masatomo Takahashi) have discovered some interesting relations of graph-like Legendrian unfoldings with caustics and Maxwel sets of corresponding Lagrangian submanifolds [2].

II) Applications to Lorentz differential geometry.

We apply the theory of wave front propagations to Lorentz differential geometry of submanifolds in Lorentz space forms (i.e., Minkowski space-time, de Sitter space or anti-de Sitter space). Since we do not have the notion of constant time in the relativity theory, we have to always consider one-parameter families of submanifolds depending on the time-parameter (i.e., world sheets). In this case, the submanifold with the constant parameter is not necessarily the constant time in the ambient space. Therefore the theory of wave front propagations is especially important for applications to Lorentz differential geometry. Of course, we can apply the theory to one-parameter family of submanifolds in Riemannian space forms (i.e., Euclidean space, the sphere or Hyperbolic space) as a spacial case. Moreover, if we have a time, I try to introduce other applications such as PDE etc.

References

[1] S. Izumiya, *Perestroikas of optical wave fronts and graphlike Legendrian unfoldings.* J. Differential Geom. **38** (1993), 485–500.

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[3] V. M. Zakalyukin, *Reconstructions of fronts and caustics depending one parameter and versality of mappings*, J. Sov. Math. 27 (1984), 2713–2735.