Geometry of singular surfaces

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In this course, we consider differential geometric study of singular points of wavefront surfaces. The talk will be separated into three parts.

1 Wavefronts and singularity

We start to introduce wavefronts and singularity. We deal with recognition problem of singularities of fronts. We give useful criteria for generic singularities of fronts, namely cuspidal edge and swallowtail, and it play a fundamental role in the other two parts. We give examples of recognition of singularities arisen from some differential geometric situations. Criteria for other fundamental singularities will be also presented.

2 Local properties of fronts

We consider differential geometric invariants of cuspidal edge. There is two fundamental invariants of cuspidal edge, it can be regarded as principal curvatures. Behavior of these invariants near swallowtail will be discussed. Moreover, the Gaussian curvature and the mean curvature diverges in general. We consider relationships between these invariants and behavior of the Gaussian and mean curvatures.

3 Global properties of fronts

We consider integration of the above invariants on compact manifolds. Then we get the Gauss-Bonnet type theorems. We give proofs of these theorems and introduce various applications of them.

References

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More detailed list of the references will be given as a handout.