How to sink a ship

School on Singularity Theory
Mini-course on Applications of Singularities

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COMPETE

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Applications of Singularities — course outline

1. Singularities of functions.
   Applications:
   - How to sink a ship
   - Catastrophe machines
2. Singularities of differential equations with two time scales.
   Application:
   - Models for nerve impulse
3. Singularities and symmetry.
   Application:
   - Deformation of an elastic cube.

Singularity Theory studies the dependence on parameters of objects.
Objects come from analysis and geometry, or physics, or from some other science.

The exact values of parameters usually (generically) do not have a big influence on the phenomena studied:
after small changes of parameter values
quantitative aspects change,
but qualitative, topological features remain the same.

For some exceptional values of the parameters
a very small variation of the parameter induces
a sudden change of qualitative features.

This change is called a bifurcation or catastrophe or perestroika, in different branches of the sciences.

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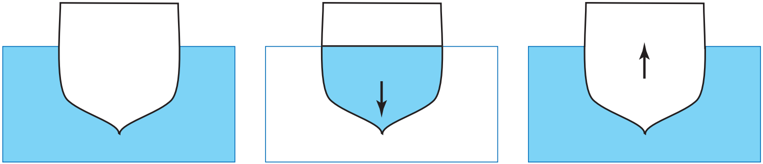
Today we'll use this to sink a ship.

How a ship floats

Archimedes principle ($\approx$ 200 B.C.)

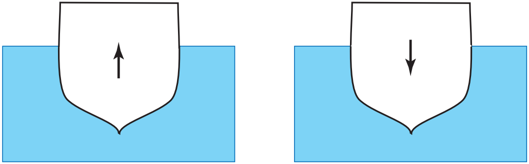
Bouyancy

Force equal and opposite to the weight of the displaced water, applied to the center of mass of the displaced water.



Forces acting on the ship

Bouyancy — Force equal and opposite to the weight of the displaced water, applied to the centre of mass of the displaced water.

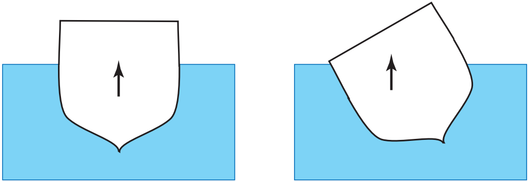


Weight — Force applied to centre of mass of the ship.

Vertical equilibrium attained at the depth for which weight and buoyancy have the same value.

The position of the centre of mass of the displaced water, when vertical equilibrium is attained is called centre of buoyancy of the ship.

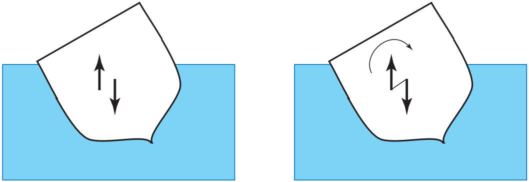
Ship



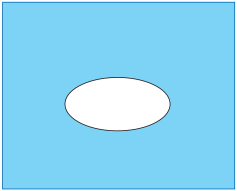
The position of the centre of buoyancy depends on the shape of the submerged part of the ship.

The shape of the submerged part depends on the angle made by the symmetry axis of the ship with the vertical.

When weight and buoyancy are not aligned this creates a torque.

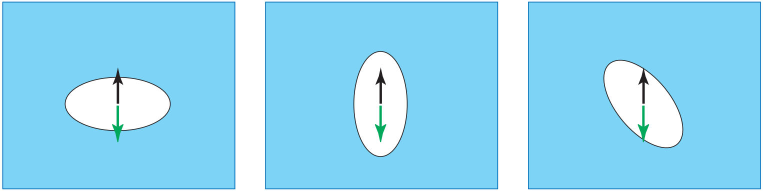


Submarine



The position of the centre of buoyancy is always the same.

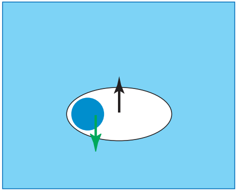
If the density of the submarine is uniform,
then the centre of mass coincides with the centre of buoyancy.



The submarine is in equilibrium in any position.

Submarine with a dense bubble

The position of the centre of buoyancy is always the same.
If the density of the submarine is not uniform, then
the centre of mass does not coincide with the centre of buoyancy.



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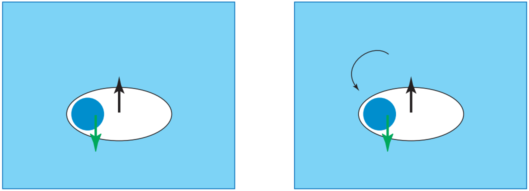

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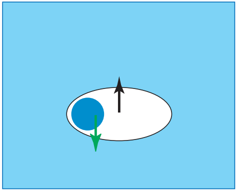
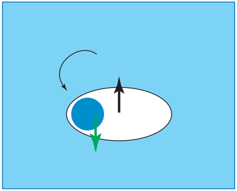
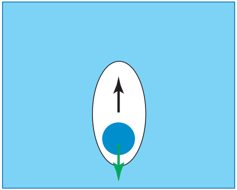
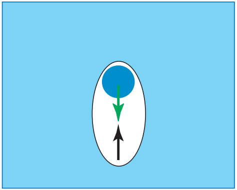




The submarine has only two equilibrium positions.

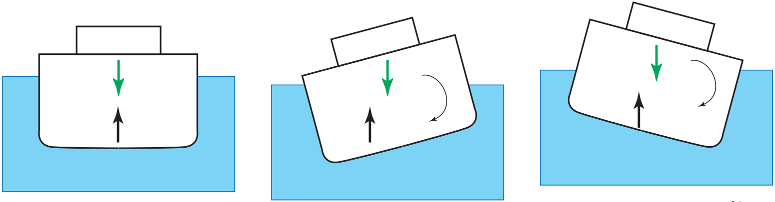


stable equilibrium



unstable equilibrium

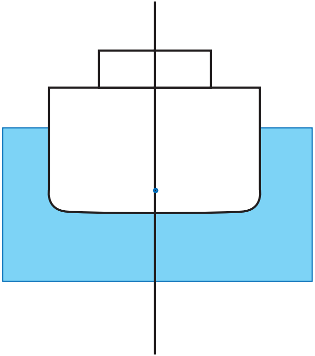
Ship



equilibrium restoring moment moment.... disastrous

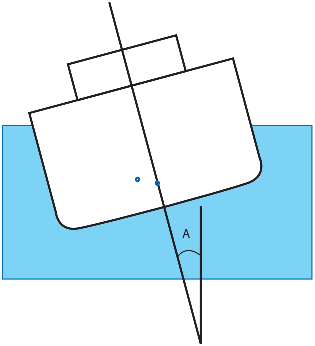
We want to determine the position of the centre of buoyancy (we will paint it on the ship).

Centre of buoyancy for the ship



To each angle A corresponds a position of the centre of buoyancy,

Centre of buoyancy for the ship



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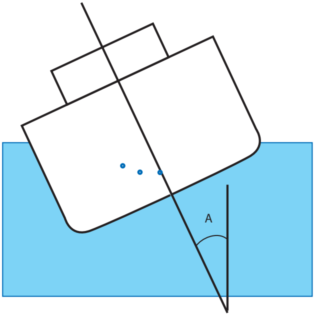

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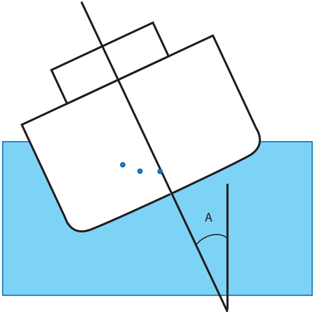
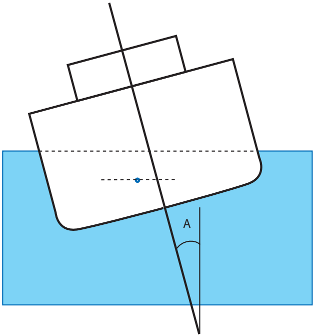


To each angle A corresponds a position of the centre of buoyancy, defining a curve $F = f(A)$.

Centre of buoyancy for the ship

When the angle A increases



Centre of buoyancy for the ship

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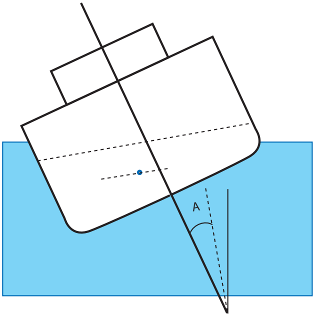

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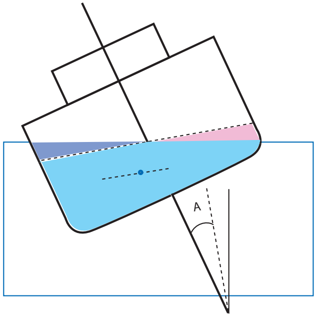
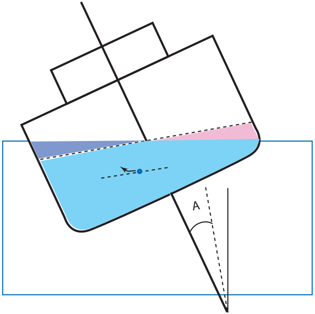


the submerged area on top increases on the slanting side,

Centre of buoyancy for the ship

When the angle A increases

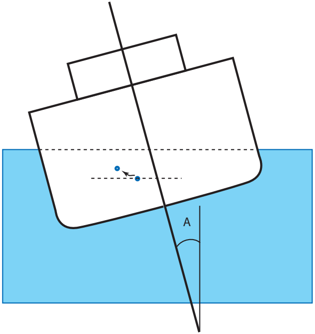


the submerged area on top increases on the slanting side,

the centre of buoyancy moves up

Centre of buoyancy for the ship

When the angle A increases

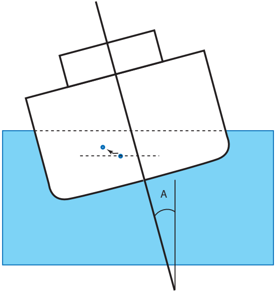


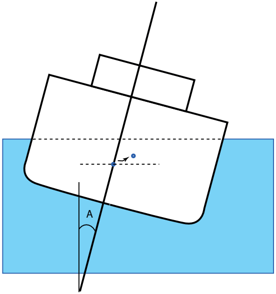
the submerged area on top increases on the slanting side,

the centre of buoyancy moves up

Centre of buoyancy for the ship

When the angle A increases the centre of buoyancy moves up.





When the angle A decreases the centre of buoyancy also moves up.

Centre of buoyancy for the ship

When the angle A increases the centre of buoyancy moves up.

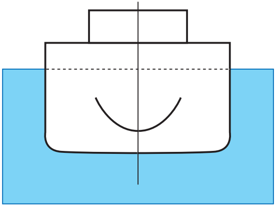
When the angle A decreases the centre of buoyancy also moves up.

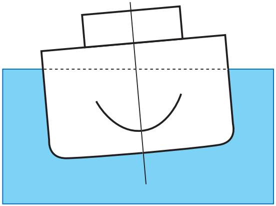
The water line is parallel to the tangent of the graph of F.

Weight and buoyancy are perpendicular to the graph of F.

Centre of buoyancy for the ship

The curve $F(A)$ of the centres of buoyancy is convex.





Equilibrium occurs at an angle A if the centre of mass is on a line perpendicular to the curve $F(A)$

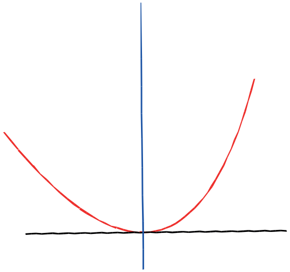
Centre of buoyancy for the ship

The curve $F(A)$ of the centres of buoyancy is convex.

Equilibrium occurs at an angle A if the centre of mass is on a line perpendicular to the curve $F(A)$
like in the gravitational catastrophe machine.

Gravitational catastrophe machine

The centre of mass should be on a line perpendicular to the parabola $F(A)$



Gravitational catastrophe machine

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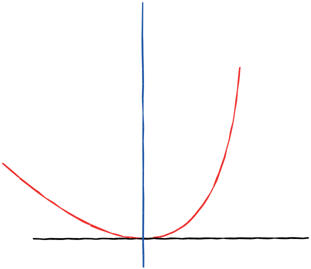

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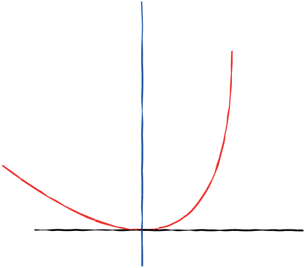

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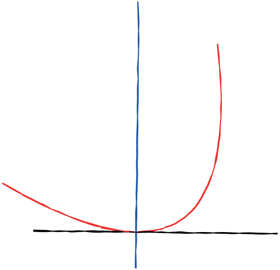

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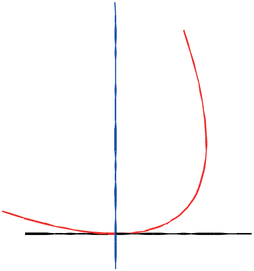

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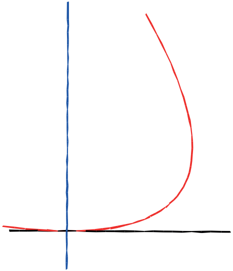
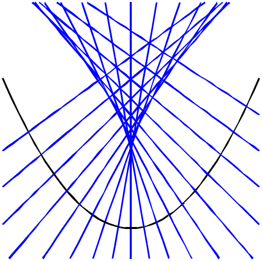


Lines perpendicular to the parabola $F(A)$



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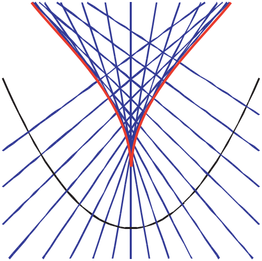
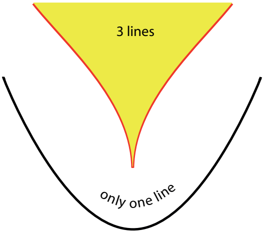
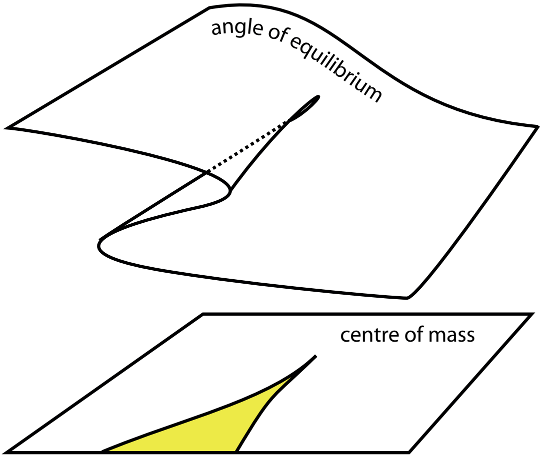


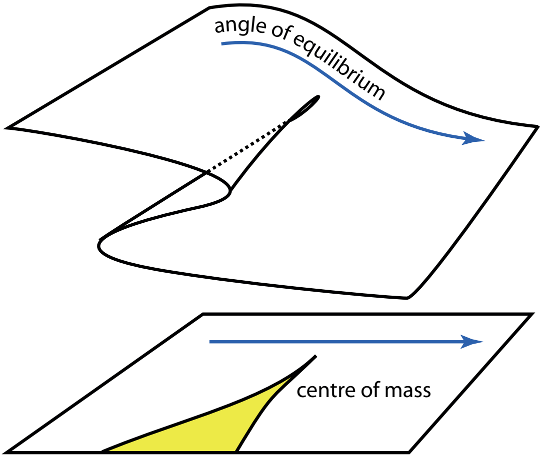
Cusp



Straight sided ship

For a “rectangular” ship, the curve $F(A)$ of the centres of buoyancy is a parabola.


angle of equilibrium
centre of mass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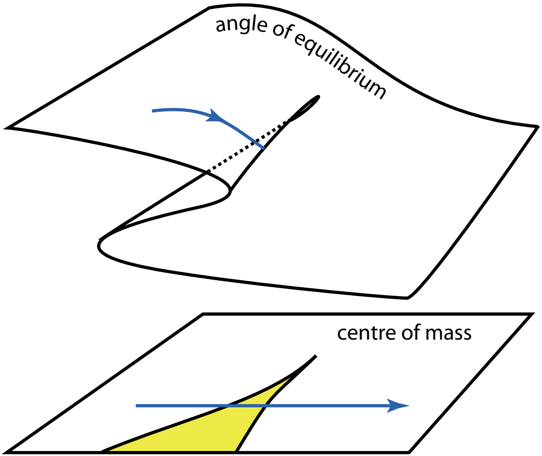
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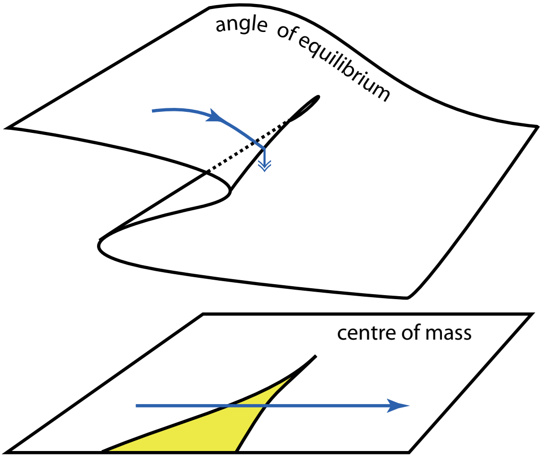
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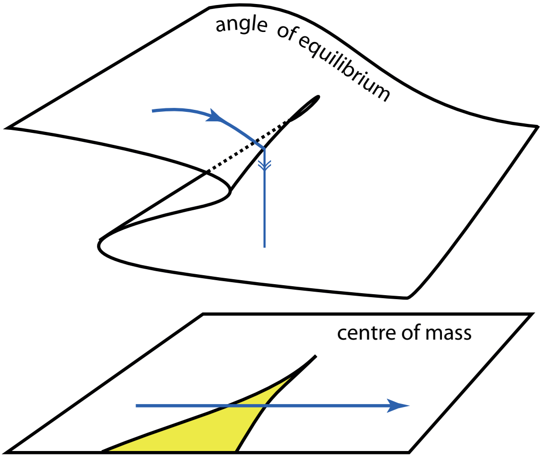
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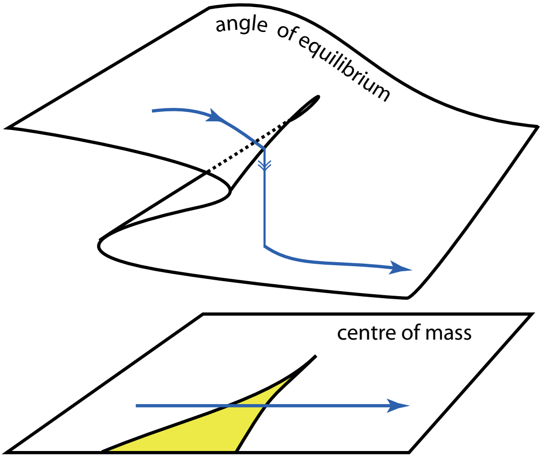
angle of equilibrium
centre of mass

More about jumps — tomorrow!

If you want to know more about ships
or about catastrophe machines, look at:

T. Poston and I.N. Stewart
Catastrophe Theory and its Applications,
Pitman 1978