18TH INTERNATIONAL WORKSHOP ON REAL AND COMPLEX SINGULARITIES

CELEBRATING THE 60TH BIRTHDAY OF
JUAN JOSÉ NUÑO BALLESTEROS
VALÈNCIA, SPAIN (JULY 21 - 26, 2024)

Book of Abstracts

VNIVERSITAT DE VALÈNCIA
Facultat de Ciències Matemàtiques

The International Workshop on Real and Complex Singularities is a key event for researchers working in Singularities Theory, Algebraic Geometry, Bifurcation Theory and related areas. It occurs biennially and attracts senior renowned researchers, young researchers and students.

Traditionally, this conference is held in University of São Paulo, São Carlos, Brazil. In 2024, we are honored to celebrate the work and 60th anniversary of Prof. Dr. Juan José Nuño Ballesteros, therefore the 18th International Workshop on Real and Complex Singularities will take place in University of Valencia, Valencia, Spain, from July 21 to 26, 2024.

This edition of the event features plenary and parallel talks, minicourse and poster presentations. The abstracts are listed below, by kind of presentation, in alphabetical order of presenters' name.

We appreciate the support of Facultat de Matemàtiques and Departament de Matemàtiques from Universitat de València.

We hope you have a nice event.

Organizing Committee

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1. Plenary Talks

TBA

André Belotto da Silva (Université Paris Cité)

The Bruce-Roberts numbers and the logarithmic characteristic varieties

Bárbara K. Lima Pereira (USP)

We give formulas for the Bruce-Roberts number $\mu_{BR}(f, X)$ when $(X, 0)$ is an isolated hypersurface singularity (IHS) and its relative version $\mu^{-}_{BR}(f, X)$ when $(X, 0)$ is an isolated complete intersection singularity (ICIS). We show that

$$\mu_{BR}(f, X) = \mu(f) + \mu(f^{-1}(0) \cap X, 0) + \mu(X, 0) - \tau(X, 0)$$

$$\mu^{-}_{BR}(f, X) = \mu(f^{-1}(0) \cap X, 0) + \mu(X, 0) - \tau(X, 0),$$

where μ and τ are the Milnor and Tjurina numbers, respectively. As a consequence of these relations we prove that the logarithmic characteristic variety of an IHS is Cohen-Macaulay and the relative one is Cohen-Macaulay for any ICIS.

This is a joint work with J. J. Nuño-Ballesteros (Universitat de Valencia, SPAIN), B. Oréfice-Okamoto, (UFSCar, BRAZIL) and J.N. Tomazella, (UFSCar, BRAZIL).

The flat geometry of lines; how to compare geometric objects

Bill Bruce (University of Liverpool)

The talk covers two (we hope not completely) separate topics. The first reports on work with Farid Tari on the flat geometry of congruences, that is the local geometry of generic 2-dimensional families of lines in $\mathbb{R}^3$ invariant under the affine group. Familiar (if not typical) examples of congruences include the set of normals to a surface and the chords of a space curve. We show that the space of lines carries an interesting affine invariant infinitesimal structure and discuss the focal set, the dual focal set and a binary differential equation associated

to any congruence. In the second half of the talk we present a naive approach to comparing the geometry of submanifolds of homogeneous spaces invariant under some Lie group of 'isometries'G. This also allows us to compare a submanifold with itself, raising some interesting technical challenges which we have not yet solved.

Map-germs of weakly finite singularity type and Artin approximation results

Dmitry Kerner (Ben Gurion University, Israel)

Map-germ of finite singularity type (i.e. contact-finite maps) possess especially nice properties. They correspond to isolated singularities. I will speak about maps of "weakly finite" singularity type. One allows non-isolated singularities that are not too pathological. The definition goes via the (higher) critical loci for maps with singular targets. This version of the critical locus (seemingly not well known) contains much more information about a map than the classical singular locus.

As a justification of the non-pathological behavior of these maps I will state Artin approximation results. Any formal left-right equivalence of such maps is approximated by ordinary (e.g. analytic or Nash) left-right equivalence.

Some aspects of polar and discriminant curves

Evelia Rosa García Barroso (Universidad de La Laguna)

We will show some recent results on polar curves and discriminant curves and related invariants.

On geometric deformations of discriminants and apparent contours

Farid Tari (University of São Paulo)

The main part of the talk is about geometric deformations of apparent contours of surfaces in the Euclidean 3-space, and more generally, of discriminants of map-germs from the plane to the plane (joint work with Mostafa Salarinoghabi and Mararu Hasegawa). Geometric deformations mean those that take into consideration the deformation of the singularity as well as of the geometry of the curve. We also define geometric invariants of the above discriminants and use the results in [J.W. Bruce, M. A. C. Fernandes and -T, On geometric invariants of singular plane curves.

arXiv:2405.19239] to obtain some of their properties.

Topological roots of the Bernstein-Sato polynomial of plane curves

Guillem Blanco (KU Leuven)

In this talk we will present a set of topological roots of the local Bernstein-Sato polynomial of arbitrary plane curve singularities. These roots are characterised in terms of certain divisorial valuations and the numerical data of the minimal log resolution. In particular, this set of roots strictly contains both the opposites of the jumping numbers in $(0, 1)$ and the poles of the motivic zeta function counted with multiplicity. As a consequence, we prove the multiplicity part of the Strong Monodromy Conjecture for $n = 2$.

Rational homology manifolds and cubical hyperresolutions

Irma Pallarés Torres (KU Leuven)

In this talk, we will introduce a characterization of rational homology manifolds in terms of cubical hyperresolutions and discuss some applications in the theory of characteristic classes. This is a joint work with J. Fernández de Bobadilla.

A metric characterization of affine linear spaces among algebraic varieties

Jose Edson Sampaio (Universidade Federal do Ceará)

In 1973, C. P. Ramanujam proved a beautiful topological characterization of $\mathbb{C}^2$ as an algebraic surface, he proved that *if X is an affine smooth complex algebraic surface that is contractible and simply connected at infinity, then X is isomorphic to $\mathbb{C}^2$ as an algebraic variety.* In the same work, Ramanujam presented an example of a 3-dimensional smooth complex algebraic variety homeomorphic to $\mathbb{C}^3$, but not isomorphic to $\mathbb{C}^3$. Thus, the characterization of Ramanujam does not hold in higher dimension. However, Ahern and Rudin (1993) showed that *a pure dimensional complex algebraic set that is C^1 smooth at infinity (i.e., the union of its inversion and 0 is C^1 smooth at 0) must be an affine linear subspace.* A. Fernandes and J. E. Sampaio (2020) proved a metric version of the result of Ahern and Rudin, they proved that *a pure dimensional complex algebraic set that is Lipschitz regular at infinity (i.e., it is bi-Lipschitz homeomorphic, outside of compact subsets, to a Euclidean space) must be an affine linear subspace.*

In this talk, we present a result that generalizes the above-mentioned results from Ahern and Rudin, and Fernandes and Sampaio. More precisely, we show that a pure dimensional complex algebraic set that is log-Lipschitz regular at infinity (i.e., the union of its inversion and 0

is bi-log-Lipschitz homeomorphic to a Euclidean ball) must be an affine linear subspace.

This result is contained in the preprint arXiv:2404.06943.

Juanjo's mathematical work

Bruna Oréfice Okamoto (UFSCar), Guillermo Peñafort-Sanchis (Universidad de Valencia) and Juan Antonio Moya-Perez (Universidad de Valência)

In this talk, we see in a very informal way part of Juanjo's work, starting with his initial results in geometry, topology of applications between smooth spaces, mappings on singular spaces and determinantal singularities.

$\mu = \tau$ Theorems on Analytic Varieties

Konstantinos Kourliouros (Imperial College London)

In this talk we give several characterisations of the equality of the Milnor and Tjurina numbers of a function f, defined on an analytic variety X. In particular, whenever these numbers are well defined and finite, we characterise their equality in terms of (a) logarithmic vector fields, (b) relative holomorphic differential forms vanishing on the smooth part of X, and (c) eigenvalues of the monodromy operator acting in the relative vanishing cohomology of a pair of Milnor fibers of f and of its restriction f_X on X. Finally we show that under certain conditions on the stratified singularity type of the pair (f, X), the following relative analog of K. Saito's theorem holds true: equality of the relative Milnor and Tjurina numbers of the function f with respect to X (also called Bruce-Roberts numbers) is equivalent to the relative quasihomogeneity of the pair (f, X), i.e. to the existence of coordinate system such that both f and X are quasihomogeneous with respect to the same system of positive rational weights.

Vanishing arcs for isolated plane curve singularities

Pablo Portilla Cuadrado (Universidad Politécnica de Madrid)

It is a classical theorem in singularity theory that the variation operator associated with an isolated hypersurface singularity is an isomorphism between the relative homology and the absolute homology of the Milnor fiber. In this talk we introduce a homotopy version of this variation operator. Using a joint result with Nick Salter we give criterions to decide if a properly embedded arc is sent to a geometric vanishing cycle by this operator. And using a joint result with Baldur Sigurdsson, we are able to find families of arcs satisfying that criterion.

Good real pictures of complex maps

Roberto Giménez Conejero (Alfréd Rényi Institute of Mathematics)

Given a holomorphic map germ $f_C : (C^n, 0) \to (C^p, 0)$, the problem we are interested in is finding a real map germ $f_R : (R^n, 0) \to (R^p, 0)$ such that its complexification is equivalent to f_C and all the topological data of f_C can be found in f_R. More precisely, one wants to find that the topology of the a generic perturbation of f_C (equivalent of the Milnor fiber for maps) is realised as a real object .

I will introduce the problem and the (new) techniques and ideas we use. We are very optimistic about this approach, so far we have a very restrictive necessary condition and a proof of a (generalization of a) conjecture of T. Cooper and D. Mond: the inclusion of the real image into the complex is a homotopy equivalence.

This is an on-going joint work with Ignacio Breva Ribes.

On the scheme structure of arc spaces

Roi Docampo (University of Oklahoma)

The arc space of an algebraic variety parametrizes formal germs of smooth curves mapping into the variety. It is an infinite dimensional scheme whose geometry can be used to analyze and control invariants of singularities. Traditionally this is done by focusing on the topological structure of arc spaces, but recent developments suggest that its scheme structure (its singularities, its non-reduced scheme structure) should also play an important role. For instance, embedding dimensions and codimensions in local rings of arc spaces are related to the computation of discrepancies and to a theorem of Drinfeld, Grinberg, and Kazhdan (DGK) on formal neighborhood of arcs. In collaboration with C. Chiu and T. de Fernex, we have been developing a toolbox for the study of the scheme structure of arc spaces and jet schemes. The starting point is an explicit formula for the sheaf of differentials on the arc space, which leads to new results (a converse to the DGK theorem, the control of arc fibers, and a description of Nash blow-ups of jet schemes) as well as simpler and more direct proofs of some of the fundamental theorems in the literature (numerical versions of the birational transformation rule in motivic integration and the Denef-Loeser lemma, and a new proof of the curve selection lemma for arc spaces). In this talk I will give an overview of the latest developments in this area.

Hilbert schemes, multi-singularities and Thom polynomials

Toru Ohmoto (Waseda University)

I will speak about Thom polynomials and Hilbert schemes, including a proof of the existence of multi-singularity Thom polynomials, known as Kazarian's conjecture [arXiv:2406.12166]

Singularities of mixed polynomials with Newton polyhedrons
Toshizumi Fukui (Saitama University)

We introduce a mixed toric modification as a mixed analogy of toric modifications and discuss when this provides an analogy of resolutions of singularities defined by mixed polynomials. A mixed toric modification is associated with a mixed fan, which is a notion we discuss. We investigate non-degeneracy condition for mixed polynomials and discuss when this provides a semi-algebraic or real algebraic analogue of resolutions of singularities.

2. Mini-course

A long lived conjecture
David Mond (University of Warwick)

This is a mini-course of three lectures.

1. In the first I will use drawings of plane curves and surfaces in 3-space to motivate the conjecture that the image Milnor number μ_I of an unstable map-germ $(\mathbb{C}^n, S) \to (\mathbb{C}^{n+1}, 0)$ is greater than or equal to its deformation-theoretic codimension (its $\mathscr{A}_e$-codimension), with equality in case f is quasi-homogeneous. This will be followed by some basic definitions and simple calculations. I run through some of the evidence for the conjecture, and introduce a closely related theorem relating the discriminant Milnor number and the $\mathscr{A}_e$-codimension of a germ $(\mathbb{C}^n, S) \to (\mathbb{C}^p, 0)$ when $n \geq p$.
2. The second lecture introduces more technical detail and begins the proof of a special case of the conjecture, concerning multi-germs of immersions. The proof focuses on the notion of Cohen-Macaulay module, which is used in many cases in Singularity Theory to prove "conservation of multiplicity"", with which one can calculate the number of special points of certain kinds emerging from an unstable singularity as it is deformed, as the vector-space dimension of certain algebras and modules. I do not assume prior knowledge of this notion. Essentially the same proof works for the theorem on discriminants referred to in the first lecture, but does not work in the general case of the conjecture.
3. The third lecture concludes the proof, and introduces a closely analogous conjecture, due to Victor Goryunov in 2021, about the vanishing

homology of matrix singularities. It ends with some speculations on alternative lines of attack for the two conjectures.

3. Parallel Sessions

On strong Euler-homogeneity for free divisors

Abraham del Valle Rodríguez (Universidad de Sevilla)

In 2002, it was conjectured that a free divisor satisfying the so-called Logarithmic Comparison Theorem must be strongly Euler-homogeneous and it was proved for the two-dimensional case. In 2006, it was shown that the conjecture is also true in dimension three, but, today, the answer for the general case remains unknown. In this talk I will give a new characterization of strong Euler-homogeneity that will allow us to deduce some partial answers to this problem. In particular, we will be able to:

- Prove a weaker version of the conjecture:

  If D is a germ of free divisor in $(\mathbb{C}^n, 0)$ satisfying LCT and it is strongly Euler-homogeneous outside 0, then it is also strongly Euler-homogeneous at 0.
- Prove the conjecture in dimension four.
- Prove the conjecture for weakly-Koszul free divisors in arbitrary dimension.

About the local geometry of hipersurfaces, in R4, with boundary.

Ana Claudia Nabarro (USP-Brazil)

Joint with Tawana Garcia Nunez

We initiate in this work the study of the local flat geometry of smooth hypersurfaces M in $\mathbb{R}^4$, with boundary, using singularity theory. This geometry is obtained by studying the contact of M with lines, planes (see Nabarro and Martins) and hyperplanes. The contact with hyperplanes (respectively lines) is measured by the singularities of the elements of the family of heightfunctions $H : M \times S^3 \to \mathbb{R}$ (resp. projections to hyperplanes by $P : M \times S^3 \to \mathbb{R}^3$).

Our aim is to give a geometric characterization of the generic singularities of these projections.

Remark: Our inspiration is that the study of hypersurfaces in $\mathbb{R}^4$ has been lately very important. "In medicine, 4D models can be used in magnetic resonance imaging, computed tomography and ultrasound. In the case of magnetic resonance imaging methods that use 4D images have proven to be effective in facilitating the diagnosis of cardiovascular diseases. They differ from previous methods, both in terms of greater accuracy in obtaining a 3D model of the heart and in their ability to

calculate blood flow in all directions. In the case of computed tomography a new scanning protocol was created to generate 4D images of the lung. Compared to previous protocols this new one has a shorter scanning time and obtains images of the entire respiratory cycle. In ultrasound applied to prenatal exams a 4D method allows the dynamic visualization of images of the fetal heart at different levels of depth and facilitates the diagnosis of congenital anomalies". See Castelo.

Castelo Castelo Filho,,A. and Bueno L. M. Aproximações de variedades definidas implicitamente utilizando técnicas de contagem e enumeração, Sociedade Brasileira de Matemática Aplicada e Computacional, 88, 2019.

Lattice cohomology of curve singularities

András Némethi (Rényi Institute of Mathematics)

I will present some recent result about the structure of the lattice cohomology of a curve singularity (e.g. how it determines the multiplicity of the germ). It is a joint work with A Kubasch and G. Schefler.

The complete intersection discrepancy of a curve

Antoni Rangachev (Institut de mathématiques de Jussieu – Paris Rive Gauche, CNRS)

In this talk I will define the local and global version of the complete intersection discrepancy of curves. In the local setting, I will show how this intrinsic invariant shows up as a correction term if one wants to generalize the Lê-Greuel multiplicity formula for the Milnor number of complete intersection curves. In the global case, I will show how one can use the complete intersection discrepancy to derive an algebraic formula for the genus of a smooth projective curve. The talk is based on joint works with A. Bengus-Lasnier, T. Gaffney, and B. Teissier.

Links of singularities of non-degenerate mixed polynomials

Benjamin Bode (Universidad Politécnica de Madrid)

A mixed polynomial $f : \mathbb{C}^2 \to \mathbb{C}$ is a complex polynomial in two variables z_1, z_2 and their conjugates $\bar{z_1}$ and $\bar{z_2}$. Oka showed that mixed polynomials that satisfy certain non-degeneracy conditions have a weakly isolated singularity at the origin. In this talk I will show that under an additional generic assumption the links in the 3-sphere that can arise as links of singularities of these types of polynomials can be completely characterised in terms of certain symmetry constraints.

A Mond-type conjecture for frontal map germs

Christian Muñoz-Cabello (Universitat de València)

The image Milnor number of a finitely determined, smooth map germ $f\colon (\mathbb{C}^n, S) \to (\mathbb{C}^{n+1}, 0)$ is defined as the number of spheres emerging in a stable deformation of f. D. Mond conjectured that the image Milnor number is greater or equal to the codimension of f, with equality if f is quasihomogeneous. The notions of image Milnor number and codimension can also be formulated within the framework of frontal map germs, giving rise to a similar conjecture. In this presentation, we shall discuss this Mond-type conjecture in the case $n = 2$ (for corank 1) and explore the case where f is a germ of wave fronts.

The Cohen-Macaulay property of double and triple point spaces

Cinzia Villa (Universitat de València)

We study recent results about Cohen-Macaulay property of double and triple point spaces and give explicit expressions for their defining ideals.

Lines of curvature and umbilics for surfaces obtained by deformations of real Milnor links

Daniel Dreibelbis (University of North Florida)

In previous work, Gómez-Gutiérrez and Sánchez-Bringas described the global foliations of asymptotic lines on compact surfaces of genus greater than one, defined in $\mathbb{R}^4$ as the links of the real part of the Milnor fibration of a family of polynomials. By sterographic projection, they obtained the foliations of lines of curvature for certain compact surfaces of genus greater than one in $\mathbb{R}^3$. In this talk we continue their line of study by considering deformations of the original class of maps. With this, we obtain examples of stable structures of lines of curvatures/umbilics for compact surfaces of any genus greater than one. This is joint work with Federico Sánchez-Bringas and Ivonne Hernández-Martínez.

Moderately Discontinuous Homology of Real and Complex Set Germs

Davi Lopes Medeiros (Universidade Federal do Ceará)

Recently, Bobadilla, Heinze, Pereira and Sampaio presented the moderately discontinuous homology, a bi-Lipschitz subanalytic invariant that is quite flexible and recover many interesting properties, such as smooth complex analytic germs, the number of irreducible components

of complex analytic germs, the embedded topological type of plane branches, the topological type of any plane curve singularity and relative multiplicities of complex analytic germs. In this talk we are going to show the intuition behind this homology, by calculating it, for the inner metric, in all real surface germs and for a broad class of real surface germs, for the outer metric. We also discuss how one can use this homology to determine the Lipschitz Normally Embedded property and the Inner Metrical Conicalness for complex analytic normal surfaces, and analogous problems for real subanalytic sets with dimension at least 2. This is a joint work with Edson Sampaio and Emanoel Souza.

Newton Non-degeneracies of Mixed Polynomials

Eder Leandro Sanchez Quiceno (UFSCar)

Newton non-degeneracies play a crucial role in algebraic geometry and singularity theory, providing essential properties for the classes that they define. A notable property explored in this context is the presence of an isolated singularity in mappings or algebraic varieties, key aspects of the Benedetti-Shiota conjecture. In this talk, we discuss various non-degeneracies of mixed polynomials in two variables, starting with Oka's (strong) non-degeneracy, and including (strong) inner non-degeneracy and (strong) partial non-degeneracy, alongside some implications. We also introduce the Σ-Newton-stable mixed polynomials and demonstrate that strong partial non-degeneracy characterizes the occurrence of an isolated singularity in these polynomials.

Joint work with Benjamin Bode.

Keywords: Newton polygon, Newton non-degeneracy, Mixed polynomial.

Acknowledgements: The research was supported by São Paulo Research Foundation (FAPESP), grants 2019/11415-3 and 2017/25902-8.

The polyhedral type of a complex polynomial map on the plane

El Hilany Boulos (TU Braunschweig)

We say that two continuous maps $f, g : X \to Y$ are topologically equivalent if there exist homeomorphisms $\varphi : X \to X$ and $\psi : Y \to Y$ such that $\psi \circ f \circ \varphi = g$. Given any positive integer d, it is known that there are finitely many topological equivalence classes of polynomial maps of degree d on the complex plane. Due to the lack of a systematic approach, their number, denoted by $\Theta_{\mathbb{C}}(d)$, is known only for the case where $d = 2$.

The *Newton tuple* of a polynomial map is the collection of Newton polytopes corresponding to the respective polynomials. It is shown that a generic map (in the open Zariski sense) with fixed Newton pair contributes to one element in $\Theta_{\mathbb{C}}(d)$. I will present results relating topological invariants of generic maps to their Newton pairs. This gives rise to a polyhedral method that produces a lower bounds on $\Theta_{\mathbb{C}}(d)$ for arbitrary d. We compute the latter by implementing a software package on OSCAR. This is a joint work with Kemal Rose.

The $\mathbb{Q}$-algebraicity problem in real algebraic geometry

Enrico Savi (Université Côte d'Azur)

Let K be a subfield of $\mathbb{R}$. We say that an algebraic subset $V \subset \mathbb{R}^n$ is K-algebraic if it can be described by global polynomial equations with coefficients in K. Denote by $\overline{\mathbb{Q}}^r$ the real closure of $\mathbb{Q}$, that is, the field of real algebraic numbers.

In 2020, Parusiński and Rond proved that every algebraic set $V \subset \mathbb{R}^n$ is homeomorphic to a $\overline{\mathbb{Q}}^r$-algebraic set $V' \subset \mathbb{R}^n$ via a strongly equisingular semialgebraic and arcwiseanalytic deformation $h_t : \mathbb{R}^n \to \mathbb{R}^n$. The latter very general result and the example of Teissier of an irrational Whitney equisingular class motivate the following open problem:

$\mathbb{Q}$-ALGEBRAICITY PROBLEM: (Parusiński, 2021) Is every algebraic set $V \subset \mathbb{R}^n$ homeomorphic to some $\mathbb{Q}$-algebraic set $V' \subset \mathbb{R}^m$, with $m \geq n$?

In general, the fact that $\mathbb{Q}$ is not a real closed field is a crucial difficulty.

The aim of the talk is to introduce the $\mathbb{Q}$-ALGEBRAICITY PROBLEM and to explain how our new approximation techniques over $\mathbb{Q}$, inspired by Nash-Tognoli and Akbulut-King results, allowed us to provide a positive answer for some classes of real algebraic sets. More in detail, we produce $\mathbb{Q}$-algebraic Nash-diffeomorphic models of nonsingular real algebraic sets, also in a relative setting, and $\mathbb{Q}$-algebraic semi-algebrically homeomorphic models of real algebraic sets with isolated singularities.

The talk is based on two works, one of those in collaboration with Ghiloni.

On the Moser's Bernstein Theorem

Euripedes Silva (IFCE)

In this talk, we prove the following version of the famous Bernstein's theorem: Let $X \subset \mathbb{R}^{n+k}$ be a closed and connected set with Hausdorff dimension n. Assume that X satisfies the monotonicity formula at $p \in X$. Then, the following statements are equivalent:

(1) X is an affine linear subspace;

(2) X is a definable set that is Lipschitz regular at infinity and its geometric tangent cone at infinity, $C(X, \infty)$, is a linear subspace;
(3) X is a definable set, blow-spherical regular at infinity and $C(X, \infty)$ is a linear subspace;
(4) X is a definable set that is Lipschitz normally embedded at infinity and $C(X, \infty)$ is a linear subspace;
(5) the density of X at infinity is 1.

Consequently, we prove the following generalization of Bernstein's theorem: Let $X \subset \mathbb{R}^{n+1}$ be a closed and connected set with Hausdorff dimension n. Assume that X satisfies the monotonicity formula at $p \in X$ and there are compact sets $K \subset \mathbb{R}^n$ and $\tilde{K} \subset \mathbb{R}^{n+1}$ such that $X \setminus \tilde{K}$ is a minimal hypersurface that is the graph of a C^2-smooth function $u\colon \mathbb{R}^n \setminus K \to \mathbb{R}$. Assume that u has bounded derivative whenever $n > 7$. Then X is a hyperplane. Several other consequences are also presented. This is a joint work with Edson Sampaio.

Height functions on a cuspidal edge with a distinguished singular curve

Fawaz Alharbi (Umm Al-qura University-Makkah-Saudi Arabia)

We classify submersions on a cuspidal edge M in $\mathbb{R}^3$ with a distinguished singular curve S in it. We use this classification to study the contact of the pair (M, S) with planes, which is measured by the singularities of the height functions on (M, S). The discriminants of the versal deformations of the submersions obtained are also described.

The relation between jumping walls and topological type of plane curves

Ferran Dachs-Cadefau (Universitat d'Andorra)

In their respective PhD theses, Järviletho and Tucker investigated the relationship between the jumping numbers of a plane curve and its topological type. For an unibranched plane curve it is possible to recover the topological type from the jumping numbers, but for the multibranched case, such option does not exist. In this talk, we present how we can recover the topological type of a tuple of plane curves and the one of its product from its associated jumping walls.

Equivariant contact equivalence applied in condensed matter physics

Gergö Pintér (Department of Theorical physics, Budapest University of Technology and Economics)

In Thom-Mather theory, map germs are usually considered up to left-right equivalence, that is, smooth change of coordinates in the source and in the target. A coarser equivalence of map germs called contact equivalence was originally introduced as an auxiliary tool to study the left-right equivalence. However, the contact equivalence is suitable to study merging processes of roots of equidimensional real map germs.

In the talk the equivariant version of contact equivalence is used in the context of condensed matter physics, in particular, for degeneracies of electronic band structures of crystalline solids. We study how physical symmetries affect the allowed merging processes of degeneracy points and the stability of these processes. Joint work with Gabriele Naselli, György Frank, Dániel Varjas, Ion Cosma Fulga, András Pályi, Viktor Könye https://arxiv.org/abs/2403.08518.

Images of maps form $(\mathbb{C}^2, 0)$ to $(\mathbb{C}^n, 0)$

Helge Møller Pedersen (Universidade Federal do Ceará)

The question "Which germs of surface singularities in $\mathbb{C}^3$ can be realized as the image of a finite map from $\mathbb{C}^2$?" was asked by Guillermo Peñafort Sanchis. Using the extensive knowledge of the topology of three manifold there exists, we are able to prove that only quotient singularities can be the image of a map from $\mathbb{C}^2$ not just in $\mathbb{C}^3$, but $\mathbb{C}^n$ for any n.

In the talk we will go through the topological arguments, discuss which groups gives the quotients, and if times permits it give example how to use the group actions to construct maps $f : (\mathbb{C}^2, 0) \to (\mathbb{C}^n, 0)$ having a given quotient singularity as its image.

Algebraic Local Cohomology for Mixed Module and Its Application to Singularity Theory

Hiroshi Teramoto (Kansai University)

We generalize the algebraic local cohomology for modules proposed by Tajima and Nabeshima to that for mixed modules and propose a computational algorithm for this generalization. For $\mathcal{A}$, $\mathcal{K}_{\mathcal{B}}$, and ϕ-equivalences, the corresponding (extended) tangent spaces form mixed modules. In this context, the algebraic local cohomology of an (extended) tangent space is related to invariants of equivalence classes, such as codimensions. We compute the algebraic local cohomologies in several examples in singularity theory and discuss their properties. This is a collaborative work with Katsusuke Nabeshima.

Augmentations for tackling conjectures of map-germs with one-parameter stable unfolding

Ignacio Breva Ribes (Universitat de València)

In the case of smooth hypersurfaces with isolated singularity in $\mathbb{C}^n$, it is well known that the Milnor number is always greater than or equal to the Tjurina number. Recently, Liu proved that the quotient μ/τ is less than or equal to n. For map-germs in $\mathbb{C}^n \to \mathbb{C}^{n+1}$, these invariants translate to the Image Milnor Number and the $\mathcal{A}_e$-codimension. Do inequalities similar to the hypersurface case hold? For the first inequality, this problem is known and Mond's conjecture; the second has not been studied yet.

In this talk, we address both conjectures for the case of map-germs admitting one-parameter stable unfolding (OPSU). In particular, we reduce the first to studying map-germs with OPSU which are not augmentations, and prove some cases of the second. We also characterise augmentations, giving a counterexample to the proof given by Houston. Finally, we talk about simplicity of augmentations, and comment on weighted-homogeneity of map-germs with OPSU.

Lipschitz and Contact Geometries of Mixed Varieties

Inácio Rabelo (ICMC-USP)

Mixed functions are real analytic functions on complex variables and their conjugates. They generalize complex functions and inherit several properties from this context. Previous works by J. Seade, M.A.S. Ruas, A. Verjovsky, M. Oka, J. Cisneros-Molina, and others have established some of these connections and properties. Our main focus is on the geometry of mixed functions and the associated varieties. More specifically, we address the problem of topological and bi-Lipschitz equivalence of singularities in a family of mixed polynomials of Pham-Brieskorn type. We derive the necessary conditions on the exponents and extend the investigation to the Lipschitz geometry of associated mixed surfaces in the case of two variables.

Global topological classification of quadratic-like foliations on the plane

Ingrid Sofia Meza Sarmiento (Universidade Federal de Itajubá)

Let $p : \mathbb{R}^2 \to \mathbb{R}$ be a smooth function such that $\partial p/\partial x$ and $\partial p/\partial y$ have no common zeros in $\mathbb{R}^2$, i.e., p is a smooth submersion. Then the Implicit Function Theorem states that p defines a regular foliation on $\mathbb{R}^2$, denoted here by $\mathscr{F}(p)$, whose leaves are the connected components of the level sets of p.

On the other hand, we say that two regular foliations $\mathscr{F}(p)$ and $\mathscr{F}(q)$ of open subsets A and B of $\mathbb{R}^2$, respectively, are *topologically equivalent* (resp. o-*topologically equivalent*) if there exists a homeomorphism (resp. orientation preserving homeomorphisms) $h : A \to B$ carrying each leaf of $\mathscr{F}$ onto a leaf of $\mathscr{G}$.

The main objective in this talk is to present local and global properties for the class of quadratic-like foliations that are given by submersion functions

$$p(x, y) = r(x) + s(x)y + t(x)y^2, \tag{1}$$

where r, s and t are polynomials in $\mathbb{R}[x]$. In fact, we completely classify the topological equivalence of these foliations in strips.

Joint with Francisco Braun (UFSCar – Brazil) and Filipe Fernandes (UnDF – Brazil).

Indices

Jean-Paul Brasselet (CNRS)

The notion of vector field index at a singular point plays a fundamental role in applications of obstruction theory. That is the case in (one of the possible proofs of the) Poincaré-Hopf theorem or in (one of the possible definitions of) Chern characteristic classes.

That is the way Marie-Hélène Schwartz used to define her classes for singular complex stratified varieties. The lecture will be a survey of these notions, which have the advantage of allowing a better understanding of the meaning of the objects considered.

Topological classification of simple Morse-Bott functions on the projective plane

Joao Carlos Ferreira Costa (UNESP)

In this work we investigate the classification of Morse-Bott functions defined on the projective plane $\mathbb{R}P^2$, up to topological conjugacy. We give a complete topological invariant of simple Morse-Bott functions $f : \mathbb{R}P^2 \to \mathbb{R}$. The invariant introduced is inspired by the Reeb graph and it is called here equipped MB-Reeb graph. This is a joint work with Erica Batista and Ingrid Meza-Sarmiento.

Kulikov models via the Minimal Model Program

José Galindo Gimenes (Universität Bonn)

A famous result from Kulikov and from Persson and Pinkham states that for a 1-parameter semistable de-generation of smooth K-trivial

surfaces there exists a K-trivial semistable degeneration birational to it known as Kulikov model. Recently, the MMP has been used to produce a weaker version of this theorem for hyper-Kähler manifolds in the work from Kollár, Laza, Saccà and Voisin. Based on this idea we give a new proof of the Kulikov-Pinkham-Persson theorem by applying the MMP to a projective degeneration in the category of complex analytic spaces. Moreover, the same arguments yield us mildly singular Kulikov models for schemes.

Kollár, János and Laza, Radu and Saccà, Giulia and Voisin, Claire, *R*emarks on degenerations of hyper-Kähler manifolds, Annales de l'Institut Fourier 68, N. 72837–2882, 2018.

Kulikov, Viktor S., *D*egenerations of K3 surfaces and Enriques surfaces, Mathematics of the USSR-Izvestiya 11, N. 5, 1977.

Persson, Ulf and Pinkham, Henry *D*egeneration of surfaces with trivial canonical bundle, Annals of Mathematics, 45–66, 1981.

A combinatorial approach to moduli spaces of line arrangements

Juan Viu-Sos (Universidad Politécnica de Madrid)

The moduli space $\mathcal{M}(\mathcal{A})$ of a line arrangement $\mathcal{A}$ in $\mathbb{CP}^2$ is the space of arrangements with the same combinatorial information as $\mathcal{A}$ (i.e. incidence relations between lines) modulo linear projective transformations. Due to a result of Randell, two arrangements in the same connected component of $\mathcal{M}(\mathcal{A})$ have homeomorphic embeddings in $\mathbb{CP}^2$.

One of the main issues in the theory of line arrangements is to explore how much information of the embedding is determined by the combinatorics (i.e. the fundamental group of the complement, betti numbers of the associated Milnor fiber, ...). Therefore, one is interested in the construction of non-connected moduli spaces.

In this talk, we will present some combinatorial techniques in order to determine topological and geometrical information of $\mathcal{M}(\mathcal{A})$ using combinatorial classes of arrangements characterized by a recursive structure, after the seminal work by Nazir and Yoshinaga in 2011.

This is a joint work with Benoît Guerville-Ballé.

Geomery on the central singularity of D_4^--bifurcation of fronts

Kentaro Saji (Kobe University)

In this talk, a Bruce-West type normal form for the central singular point of the D_4^--bifurcation of fronts will be given. All of the coefficients of the form are differential geometric invariants of the singular point. Using this property, differential geometric studies of this singularity

will be presented. A global formula for fronts with this singular points will also be given.

Real zeros of nonnegative polynomials and applications

Khazhgali Kozhasov (Université Côte d'Azur)

Real zeros of a nonnegative polynomial $f \in \mathbb{R}[x, y]$ are singular points of the plane curve $\{f = 0\} \subset \mathbb{C}^2$. Their total number and singularity type sometimes allow to distinguish f from being a sum of squares (which are particular nonnegative polynomials). Thus, for example, a sum of squares of degree $2d$ cannot have more than d^2 real zeros (provided this number is finite), while there exist nonnegative polynomials of (large) degree $2d$ with at least $4d^2/3$ many real zeros. I will discuss various results around real zeros of nonnegative polynomials, show several applications and state some open questions.

A Thom Isotopy Theorem for nonproper semialgebraic maps

Luis Renato Gonçalves Dias (IME - UFU)

We prove a version of the Thom Isotopy Theorem for nonproper semialgebraic maps $f : X \to \mathbb{R}^m$, where $X \subset \mathbb{R}^n$ is a semialgebraic set and f is the restriction to X of a smooth semialgebraic map $F : \mathbb{R}^n \to \mathbb{R}^m$.

A class of non-normal toric surfaces in $\mathbb{C}^4$ with isolated singularity

Maria Elenice Rodrigues Hernandes (Universidade Estadual de Maringá)

In the study of normal toric surfaces $X(S) = \mathcal{V}(I_S)$, O. Riemenschneider (1981) explicitly calculated a set of minimal generators for I_S using continued fractions. In this talk, we consider a toric surface in $\mathbb{C}^4$ generated by the semigroup $S = \{(1, 0), (\lambda, 1), (0, n), (0, m)\}$ with $\lambda, n, m \in \mathbb{N}^*$, $1 < n < m$, and $gcd(n, m) = 1$. In this case, $X(S)$ has an isolated singularity and smooth normalization.

Our purpose is to present a set of generators of the binomial ideal I_S. In some sense, the sucessive division method applied to n and m provides us the exponents and the number of binomials in I_S. This is a joint work with Maria Aparecida Soares Ruas and Thaís Maria Dalbelo.

On generalised framed surfaces in the Euclidean 3-space

Masatomo Takahashi (Muroran Institute of Technology)

We have introduced framed surfaces and one-parameter families of framed curves as smooth surfaces with singular points. The framed surface is a surface with a moving frame based on the unit normal vector of the surface. Thus, the notion of framed surfaces (respectively, framed base surfaces) is locally equivalent to the notion of Legendre surfaces (respectively, frontals). A more general notion of singular surfaces, called generalised framed surfaces, is introduced. The notion of generalised framed surfaces includes not only the notion of framed surfaces, but also the notion of one-parameter families of framed curves. It also includes surfaces with corank one singularities at least locally. We investigate properties of generalised framed surfaces.

This is joint work with Haiou Yu.

Classification of algebraic surfaces and curves via deformations and special singularities

Meirav Amram (SCE)

We classify algebraic curves and surfaces via deformations to planar and non-planar shapes. In these shapes, singularities with high multiplicities appear. It is interesting to study them and to see what they contribute to computations of fundamental groups, that are invariants of the classification by themselves.

Indefinite Improper Affine Spheres and Their Singularities

Michal Zwierzynski (Warsaw University of Technology)

Francisco Milan explored the affine Björling problem, proving that, under certain conditions, an improper affine sphere (IAS) can be uniquely reconstructed from a part of its singular set. In this talk, we will demonstrate how his theorems can be extended, examine new types of singular points in a generic IAS, and introduce new Gauss-Bonnet type formulas that account for recently identified corank 2 singular points.

Constructing Gutierrez-Sotomayor flows: a few examples

Murilo Zigart (UNICAMP)

First, we explore the local construction of isolating blocks for the singularities of Gutierrez-Sotomayor flows, using tools from Conley Index Theory such as Lyapunov functions and the Conley index. We restrict this study to dimension two, where the local charts for the singularities of a Gutierrez-Sotomayor flow are an open disk, a cone, a cross-cap, and the intersection of two or three planes in general position. After that, we talk about the topological obstruction for gluing the isolating

blocks in order to construct a global flow on a closed singular manifold. We end the discussion presenting a few theorems and examples.

Topology of functions with non-isolated stratified critical points

Nicolas Dutertre (Université d'Angers)

Let $f : (\mathbb{R}^n, 0) \to (\mathbb{R}, 0)$ be a definable function germ of class $\mathcal{C}^2$ and let $(X, 0) \subset (\mathbb{R}^n, 0)$ be a germ of a closed definable set. We investigate topological invariants associated with $f_{|X}$. In particular, we give several topological formulae for the Euler characteristics of related sets. We also relate the topology of $f_{|X}$ to the topology of a definable function with isolated critical point in the stratified case. Joint work with Juan Antonio Moya Pérez (Valencia).

On vertices of frontals in the Euclidean plane

Nozomi Nakatsuyama (Muroran Institute of Tecnology)

We investigate vertices for plane curves with singular points. As plane curves with singular points, we consider Legendre curves (respectively, Legendre immersions) in the unit tangent bundle over the Euclidean plane and frontals (respectively, fronts) in the Euclidean plane.

We define a vertex using evolutes of frontals. After that we define a vertex of a frontal in the general case. It is also known that the four vertex theorem does not hold for simple closed fronts. We give conditions under which a frontal has a vertex and the four vertex theorem holds for closed frontals. We also give examples and counter examples of the four vertex theorem.

This is joint work with Masatomo Takahashi.

Simplifying generic smooth maps to the 2-sphere and to the plane

Osamu Saeki (Kyushu University)

We study how to construct explicit deformations of generic smooth maps from closed n–dimensional manifolds M with $n \geq 4$ to the 2–sphere S^2 and show that every smooth map $M \to S^2$ is homotopic to a C^∞ stable map with at most one cusp point and with only folds of the middle absolute index. Furthermore, if n is even, such a C^∞ stable map can be so constructed that the restriction to the singular point set is a topological embedding. As a corollary, we show that for $n \geq 4$ even, there always exists a C^∞ stable map $M \to \mathbf{R}^2$ with at most one cusp point such that the restriction to the singular point set

is a topological embedding. If time permits, we give an application to open book structures on odd dimensional manifolds.

On Zariski multiplicity conjecture for quasihomogeneous surfaces with non-isolated singularities

Otoniel Nogueira da Silva (Universidade Federal da Paraíba)

In this talk, initially we consider an $\mathcal{A}$-finite map germ f from $(\mathbb{C}^2, 0)$ to $(\mathbb{C}^3, 0)$. In this case, the double point curve $D(f)$ plays a fundamental role in studying the topology of the image of f. When f is quasihomogeneous and has corank 1, we present a characterization of the fold components of the double point curve $D(f)$. As an application of this result, in our setting, we also consider Zariski multiplicity question for a pair of germs of surfaces $(X_1, 0)$ and $(X_2, 0)$ in $(\mathbb{C}^3, 0)$ with 1-dimensional singular set $(\Sigma(X_i), 0)$. Finally, we characterize the Whitney equisingularity of an unfolding $F = (f_t, t)$ of f through the constancy of the Milnor number of a certain plane curve in the source. This gives in some sense an answer to a question by Ruas in 1994.

The miniversal deformation of some complete intersection monomial curves

Patricio Almirón (Instituto de Matemáticas Universidad de Granada)

A monomial curve singularity is essentially the geometric representation of the semigroup algebra associated to a numerical semigroup. Complete intersection monomial curves (CIMC) are one of the richest examples when referring to the interplay between geometry of curve singularities and the combinatorics of the semigroup of values defined by the set of the possible intersection multiplicities with the curve. In 1976, Delorme provided an extremely useful combinatorial characterization of numerical semigroups whose semigroup algebra is a complete intersection (and hence its monomial curve). This characterization has been extensively used in the literature of numerical semigroup theory and semigroup algebras but surprisingly not used in the geometric context.

The main aim of this talk is to show how Delorme's characterization can be used to study the miniversal deformation of a CIMC. The main part of the talk will focus precisely in this connection. Concretely, we will show that we can provide a surprising general decomposition result of a basis of the miniversal deformation of any CIMC. As a consequence, we can explicitly calculate this basis for some notable families of CIMC.

An important topic related to the study of the base space of the miniversal deformation of a monomial curve is its connection with the moduli space of projective curves with a given Weierstrass semigroup.

In 1974, Pinkham showed that the dimension of the negatively graded part of the miniversal deformation is related to the dimension of such a moduli space. If time permits, we will show how our explicit computation of the basis of the miniversal deformation yields some estimates for the dimension of the moduli space of the family.

The talk is based in a joint work with J.J. Moyano Fernández.

Special 2-flags: classification in length 5 is finite

Piotr Mormul (University of Warsaw, Warsaw, Poland)

Distributions generating special multi-flags were started by Antonio Kumpera and Jacques Rubin in the year 1999. Such distributions generalize classical Goursat flags: the members of the derived flag grow in ranks regularly, always by m, $m \geq 2$. (They grow always by 1 in the Goursat case.) So the sequence of linear dimensions, the so-called big growth vector, is $[m+1,\, 2m+1,\, 3m+1, \ldots,\, m+1+rm]$ at every point of the underlying manifold of dimension $m+1+rm$. The integer r is the *length* of a flag.

Of particular interest are special 2-flags featuring at every point the big growth vector $[3,\, 5,\, 7, \ldots,\, 3+2(r-1),\, 3+2r]$. The tree of their singularities is much richer than in the Goursat theory, while their precise local classification is less advanced compared to Goursat. All existing singularities of special 2-flags materialize on the stages of the Special 2-Flags Monster Tower (S2FMT for short).

In the year 2010, in the arXiv preprint 1011.1763 by Pelletier and myself, that classification has been advanced up to flag's length 4. In the year 2020 we could only repeat that same information, summarized in the following table

length	# sing classes	# RV classes	# orbits
2	2	2	2
3	5	6	7
4	14	23	34
5	41	98	?
6	122	433	??
7	365	1935	∞

Of importance for the **present** abstract is a single question mark '?' standing for the number of orbits of the local classification in length 5. In the year 2020 Pelletier and myself gave recursive formulas for the *infinitesimal symmetries* of special 2-flags. When recently implemented by Andrzej Weber, those infinitesimal symmetries got expressed explicitly in terms of the vector fields in the 3-dimensional base of the S2FMT. The obtained formulas, when put to work in the classification problem, indicated a possible appearance of moduli in certain four (out of altogether 41) singularity classes in length 5.

Yet a detailed analysis of *all* symmetries (and not only those embeddable in the flows of the infinitesimal symmetries) showed that the four hypothetical moduli were only apparent. And such is the message of the present communication: **the local classification of special 2-flags in length 5 is finite**. However, the exact number of existing local geometries thereof (a three-digit number) is not yet precisely ascertained. While there does persist the **double** question mark in the table above concerning the next length 6.
On the other hand it is long since known that in length 7 a numerical modulus of the local classification exists (proved still in the arXiv 1011.1763). For the special 2-flags it is likely that the treshold length of the onset of moduli is 6. (For comparison, for Goursat flags such a treshold length of the onset of moduli of the local classification is 8.)

Bifurcation of Limit Cycles in Piecewise Quadratic Kolmogorov systems

Regilene Oliveira (ICMC-USP, São Carlos)

The 16th Hilbert Problem aims to find an upper bound for the number of limit cycles that a planar polynomial vector field of degree n can present. This problem as well as the local cyclicity problem were initial proposed for smooth vector fields and later extended to the piecewise smooth case. Since many natural phenomena can be modeled using piecewise differential systems, a significant interest in the study of this class of systems has emerged in past years. Electrical and mechanical systems, control theory, and even genetic networks, among others are examples where piecewise differential systems are applied. The dynamics of polynomial piecewise differential systems are richer than smooth systems of the same degree. For example, considering linear differential systems, it is well known that they do not have limit cycles. For piecewise linear systems with two zones separated by a straight line, there are examples having at least three limit cycles.

The class of planar quadratic differential systems with two transversal invariant straight lines is a subclass of the autonomous planar system called Kolmogorov system. Although there are many works about the dynamics of Kolmogorov systems, few of them investigate the bifurcation of limit cycles in the class of the piecewise quadratic Kolmogorov systems, here denoted by $\mathcal{K}_2$. In this talk (poster) we prove the existence of piecewise Kolmogorov quadratic systems separated by a straight line having at least six limit cycles. Another contribution of this work is in an extension of the classical center focus problem to non-smooth planar systems in $\mathcal{K}_2$. More specifically, we study the stability of monodromic equilibrium point on the separation curve Σ, the order of weakness and number of crossing limit cycles that bifurcate

from them. We recall that a crossing limit cycle is an isolated period orbit which intersect both zones defined by the separation curve Σ.

Equivalence on (finite) map germs $(\mathbb{R}^n, 0) \to \mathbb{R}^n$: Finiteness results and invariants between $[C^o - \mathcal{A}, Lip - \mathcal{A}]$ orbits.

Rodrigo Mendes Pereira (International Integration University of Afro-Brazilian Lusophony)

Finiteness or tameness property of a classification problem is a fundamental question in singularity theory. The question can be posed in the following way: Take a classification problem, with respect to an equivalence relation (for example, topological, analytical, Lipschitz equivalence of maps). Let us apply this relation to the set of polynomial maps, or polynomial maps of a bounded degree. In the last category, the problem is called tame, or the problem with finiteness property, if the set of equivalence classes is finite. In a polynomially bounded o-minimal structure tameness or finiteness property means that any finite dimensional definable family of the definable maps has finitely many equivalence classes (this equivalence relation is known to be no tame (by counter-examples given by Henry and Parusinski). In this talk, we discuss $\mathcal{A}$-equivalence of finite map germs (fixing a $C^o - \mathcal{A}$- class). We explore intermediate equivalence relations inside $[C^o - \mathcal{A}$,Lip-$\mathcal{A}]$, culminating with an $\mathcal{A}$-Lipschitz criterion in terms of Multi-Lipschitz contact equivalence.

Geometry on deformations of S_1 singularities

Runa Shimada (Kobe university)

In this talk, for a one parameter deformation of the S_1 singularity in the three space, a form using only isometric maps of the target will be given. Using this form, differential geometric properties of this singularity and the Whitney umbrellas that appear in the deformation will be presented.

On the Classification of Doodles by Concordance

Ryosuke Ota (Joint Graduate School of Mathematics for Innovation, Kyushu University)

A doodle is a generic immersion of the circle into the sphere. The classification of doodles as generic immersions has been studied; for example, Arnold defined three types of invariants, each related to three types of nongenericities of immersions: triple point, direct self-tangency, and indirect self-tangency. Doodles are associated with map germs from $\mathbb{R}^2$ to $\mathbb{R}^3$. Moya-Pérez and Nuño-Ballesteros studied some

one-parameter families of map germs $(\mathbb{R}^2, 0) \to (\mathbb{R}^3, 0)$ using the triviality of concordance of doodles, where concordance is a weaker equivalence relation of doodles that allows additional Morse-type singularities. In this talk, we focus on the classification of doodles by concordance.

Fiber singularities of continuous maps

Takahiro Yamamoto (Tokyo Gakugei University)

For a C^∞ map $f\colon M \to N$ between C^∞ map, a *fiber* over $q \in N$ is the map germ $f\colon (M, f^{-1}(q)) \to (N, q)$ along the inverse image $f^{-1}(q)$. The notion of fibers of c^∞ maps was introduced by Saeki in 2004. Note that to define fibers of maps we do not need smoothness for the maps. In this talk, for a continuous map $g\colon X \to Y$ between C^0 manifolds, the notion of *fiber singular points* of g, is introduced by using the notion fibers. Then, we show there is a Pignoni type formula for a simplicial map $g\colon (X, \mathcal{T}) \to \mathbb{R}^2$ of a closed surface with a triangle decomposition.

Straight line families in the plane and their envelopes

Takashi Nishimura (Yokohama National University)

There is a widespread method to represent the envelope when a given hyperplane family creates an envelope. However, one sometimes encounters cases when the widespread method fails to represent the desired envelope precisely, and is confused. At the same time, one wants to find a correct method to draw the envelope precisely.

In this talk, focused on straight line families in the plane, an easy to understand explanation is given on the recently discovered correct method to represent the envelope precisely. Moreover, I would like to explain when and why the widespread method fails to represent the precise shape of envelope as well.

Poincaré completions and singular spaces

Timo Essig (Kiel University, Germany)

Resolution of singularities via blowing up is a potent and prominent technique in singularity theory. This talk introduces a purely topological alternative: Poincaré completions of Banagl's intersection spaces.

The first are a minimal procedure for transforming spaces into Poincaré spaces by gluing cells. I will present conditions under which this transformation can be achieved by gluing only a single cell. Banagl's intersection spaces result in a spatial alternative to Goresky and MacPherson's intersection homology of singular spaces.

After introducing the technique, I will compare it to classical resolutions of singularities, highlighting the advantages and potential applications of Poincaré completions.

Singularities of the Gauss map components of a surface in $\mathbb{R}^4$ and contact with holomorphic curves.

Wojciech Domitrz (Warsaw University of Technology)

The Gauss map of a generic immersion of a smooth, oriented surface into $\mathbb{R}^4$ has two components that take values on the sphere. We study the singularities of the components of the Gauss map and relate them to the geometric properties of the generic immersion. Moreover, we prove that the singularities are generically stable, and we connect them to the contact type of the surface and J-holomorphic curves with respect to an orthogonal complex structure J on $\mathbb{R}^4$. We get the geometric interpretation of our results in terms of the second fundamental forms, cubic and quatric forms of the surface and the J-holomorphic curve. This is a joint work with Lucía Ivonne Hernández-Martínez and Federico Sánchez-Bringas.

Singularities and geometric topology of evolving flat frontal surfaces

Yanlin Li (Hangzhou Normal University)

In this talk, I will introduce the developments and applications of singularity theory in evolving flat frontal surfaces. I will discuss the singularities of evolving developable frontals. The geometric topology properties of evolving flat frontal surfaces will also be presented in this talk. This is a joint work with Zhichao Yang.

4. Posters

Approximating Milnor fibres of complex plane curves

Arthur Garcia Tonus

In the program "Jornadas de Pesquisa em Matemática" (Research Journeys on Mathematics), from the 2024 Summer Program on Mathematics of ICMC, the project "Aproximando superfícies em alta dimensão" (Approximating surfaces on high dimensions) was developed with a group of undergraduate students, consisting on visualization of projections of manifolds defined as a level set of some differentiable map. We developed a Continuation Simplex algorithm that is a refinement of Antonio Castelo's work, using the methods of Jean-Daniel Boissonnat. The result is a relatively efficient algorithm that produces

a piecewise linear manifold *isotopic* to the original manifold, which can be projected on the three-dimensional space and plotted on the computer using an appropriate software.

The aim of this work is to study the topology of Milnor fibres of holomorphic functions $f\colon \mathbb{C}^2 \to \mathbb{C}$. We intend to explore this algorithm to obtain topological information on the Milnor fibre.

Injectivity of polynomial maps and foliations in the real plane

Bruna Oréfice-Okamoto (UFSCar)

We develop tools to count the connected components of the fibers of a polynomial submersion in two real variables p. As a consequence, we get a necessary condition for a real number to be a bifurcation value of p. We further present new methods to verify that p has no Jacobian mates. These results are applied to prove that a polynomial local self-diffeomorphism of the real plane having one coordinate function with degree less than 6 is globally injective.

Joint work with F. Braun and F. Fernandes.

A Homotopical Approach to Generalized Gutierrez-Sotomayor Singularities

Dahisy Lima (Federal University of ABC)

We address the advances in the study of Gutierrez-Sotomayor singular flows (GS flows, for short) on closed singular 2-manifolds M. Moreover, we explore the dynamical behaviour of GS-singularities within GS-flows through homotopical perturbations to achieve a "minimal flow" into the class of homotopy equivalent spaces. This is done by describing the qualitative aspects of the flow in terms of a chain complex generated by the singularities of the flow and establishing a homotopical cancellation theory of GS-singularities.

A surgery formula for the topological Poincaré series

György Tötös (Babes-Bolyai University, Romania)

We consider the topological Poincaré series associated with a resolution (or plumbing) graph Γ of a normal surface singularity with integral homology sphere link. Following the splice construction we "split" Γ into certain induced graphs and consider their Poincaré series as well.

We prove that, after a suitable change of variables, these series can be connected by a surgery formula. In particular, this induces a surgery formula for the polynomial part of the Poincaré series and a new surgery formula for the Seiberg-Witten invariant of the link as well. The idea

can be extended to the rational homology sphere link case which is a work in progress.

Non-singular extensions of stable fold maps from surfaces into the plane

Koki Iwakura (Kyushu University)

For a given map, can it be extended to a submersion? This problem, known as the non-singular extension problem, will be the focus of this talk. In this talk, we specifically examine this problem for stable fold maps from surfaces into the plane. We will explain the strategy to address this issue.

A Hilbert basis for a class of non-compact groups in invariant theory

Leandro Nery de Oliveira (UFSCar)

It is known that there exist Hilbert bases for the ring of invariant polynomials under the action of a compact or reductive group on Euclidean spaces. In this work, we show a class of non-compact and non-reductive groups acting on Minkowski space, whose ring of invariant polynomials under such action possesses a Hilbert basis. This work was done in collaboration with Prof. Dr. Miriam Manoel.

Helicoidal surfaces of frontal in the Euclidian space

Luciana de Fátima Martins (IBILCE-UNESP-São José do Rio Preto)

We investigate helicoidal surfaces in the Euclidean space with profile curves being frontal. These surfaces are framed surfaces under some condition. We give basic invariants and curvatures by using the curvatures of Legendres curves. Moreover, we study parallel and focal surfaces of helicoidal surfaces. This is a joint work with Samuel P. Santos. Work supported by Fapesp 2002/10370-9 and Program CAPES-Print.

Generalized Bruce-Roberts Number for Complete Intersections

Michelle Morgado (Universidade estadual Paulista (UNESP))

This work investigate the topology of the Milnor fiber of map germs $\underline{f} : (\mathbb{C}^n, 0) \to \mathbb{C}^p$, with $n \geq p$ and $f^{(k)} := (f_1, \cdots, f_k)$ is a complete intersection for all $k = 1, \ldots, p$, restricted to a germ of variety $(X, 0) \subset (\mathbb{C}^n, 0)$. We define new numbers in this context, $\tilde{\mu}(\underline{f})$ and $\tilde{\mu}(\underline{f}, X)$, where the first one is the second number when X is $\mathbb{C}^n$. In

the case $p = 1$ and $\underline{f}$ has isolated singularity, the first number is the Milnor number by Hamm and the second number is the Bruce-Roberts number. Moreover, in this more general set-up, we obtain a formula that relates these two numbers.

On reflection graphs from n-space to n+1-space.

Milena Barbosa Gama (Universidade Federal da Paraíba (PB))

In this work we study some properties of reflected graph map germs. A reflected graph is a particular case of a reflection map, which is defined using an action of a reflection group G. Initially we consider a finite reflected graph f from $\mathbb{C}^n$ to $\mathbb{C}^{n+1}$. We present a description of the presentation matrix of $f_*\mathcal{O}_n$ as a module $\mathcal{O}_{n+1}$ in terms of the action of the associated reflection group G. As a consequence we also give a description for the defining equation of the image of f. Finally, we present an upper (and also a lower) bound for the multiplicity of the image of f.

The analytic equivalence on curves in $\mathbb{C}^3$ of multiplicity 4

Welinton Anderson Rocha (Universidade Estadual de Maringá)

In this poster we are going to show some strategies to obtain a normal form of a class of parametrized curves in $\mathbb{C}^3$ of multiplicity 4, using the $\mathcal{A}$-equivalence relation. To do so, we will analyze two sets of differentials, one for each non-monomial coordinate, and determine which powers can be eliminated from each coordinate, highlighting the interaction between coordinates, as changes in one can reintroduce terms in the other.

Weighted homogeneous surface singularities and numerical semigroups

Zsolt Baja (Babes-Bolyai University)

We consider numerical semigroups which can be represented by weighted homogeneous complex surface singularities with rational homology sphere link. It is known that this link has a Seifert fibered structure and a good plumbing presentation is given by a star-shaped graph. Previous works of László and Némethi have provided a formula for the Frobenius number of these semigroups, which raised the intriguing question of whether any numerical semigroup can be represented in this way. Based on this foundation, we developed a characterization of representable semigroups through a special class called flat semigoups. In

particular, using toric geometry we can further characterize the generators as well in case of those semigroups which associated representative has a star-shaped graph with only two legs.