

# Topological classification and stems of co-rank two map germs from the plane to the plane.

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## Abstract

The main purpose of this work is to describe the topological orbits which are in a  $\mathcal{K}$ -orbit of finitely determined map germs from  $\mathbb{C}^2$  to  $\mathbb{C}^2$ . At least for a very large number of co-rank two  $\mathcal{K}$ -orbits. The co-rank one case is described by Gaffney and Mond in [1, Proposition 4.5. and Theorem 4.6].

The simplest case of  $\mathcal{A}$ -finitely determined co-rank two map germs is the  $\mathcal{K}$ -class  $\mathcal{K}(xy, x^2 + y^3)$ , Gaffney and Mond in [1, Example 5.11] showed that there exists only one topological orbit in this  $\mathcal{K}$ -class. The next example given by Gaffney-Mond in [1, Example 5.12] is the  $\mathcal{K}$ -class  $\mathcal{K}(xy, x^3 + y^4)$ , in this case they expected that the number of topological orbits was finite.

To our surprise, we found a special type of germs in this  $\mathcal{K}$ -class that are not  $\mathcal{A}$ -finitely determined, moreover from these germs we showed that there exists a non finite number of  $\mathcal{A}$ -finitely determined germs in this  $\mathcal{K}$ -class which belong to different topological orbits. These special germs are called *stems* by D. Mond in [2] and are well known in the class of germs of maps from surfaces to 3-space, the germs  $S_\infty$ ,  $B_\infty$  and  $H_\infty$ .

Therefore, the most natural step in this work was to search for stems in other  $\mathcal{K}$ -classes. Following this point of view we give a complete answer for this question for any given co-rank two  $\mathcal{K}$ -class with finitely determined normal form. We show how to obtain stems in any  $\mathcal{K}$  class  $(xy, x^a + y^b)$ , the only exceptions are the cases (2, 3) and (2, 5).

Moreover, from the stems in these  $\mathcal{K}$ -classes we can show that there exists a non finite number of topological orbits. Again the the only exceptions are the cases (2, 3) with one topological orbit and (2, 5) with two topological orbits.

## References

- [1] Gaffney T. and Mond D., *Weighted homogeneous maps from the plane to the plane*. Math. Proc. Camb. Phil. Soc., vol. 109, 451–470, 1991.
- [2] Mond D. *Some Remarks on the Geometry and classification of germs of maps from surfaces to 3-space*. Topology **26**, 3, 1987, 361–383.